FUZZY SETS IN ≤–HYPERGROUPOIDS

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To the memory of Professor Kiyoshi Iséki

Abstract

This paper serves as an example to show the way we pass from ordered groupoids (ordered semigroups) to ordered hypergroupoids (ordered hypersemigroups), from groupoids (semigroups) to hypergroupoids (hypersemigroups). The results on semigroups (or on ordered semigroups) can be transferred to hypersemigroups (or to ordered hypersemigroups) in the way indicated in the present paper.

1 Introduction and prerequisites

An ordered groupoid (: *po*-groupoid) is a nonempty set S endowed with an order "$\leq$" and a multiplication "$\cdot$" such that $a \leq b$ implies $ca \leq cb$ and $ac \leq bc$ for every $c \in S$. Given a set S, a fuzzy subset of S (or a fuzzy set in S) is, by definition, an arbitrary mapping of S into the closed interval $[0, 1]$ of real numbers (Zadeh). Fuzzy sets in ordered groupoids have been first considered in 2002 in Semigroup Forum [7], where the following concepts have been introduced and studied: A fuzzy subset f of an ordered groupoid $(S, \cdot, \leq)$ is called a *fuzzy left* (resp. *fuzzy right*) *ideal* of S if (1) $x \leq y$ implies $f(x) \geq f(y)$ and (2) if $f(xy) \geq f(y)$ (resp. $f(xy) \geq f(x)$) for every $x, y \in S$. It is called a *fuzzy ideal* of S if it is both a fuzzy left ideal and a fuzzy right ideal of S. A fuzzy subset f of a groupoid (or an ordered groupoid) S is called a *fuzzy subgroupoid* of S if $f(xy) \geq \min\{f(x), f(y)\}$ for all $x, y \in S$. A fuzzy subset f of an ordered groupoid S is called a *fuzzy filter* of S if (1) $x \leq y$ implies $f(x) \leq f(y)$ and (2) if $f(xy) = \min\{f(x), f(y)\}$ for all $x, y \in S$. A fuzzy subset f of a groupoid S is called *fuzzy prime* if $f(xy) \leq \max\{f(x), f(y)\}$ for all $x, y \in S$. For a groupoid S and a fuzzy subset f of S, the complement of f is the fuzzy subset $f' : S \to [0, 1]$ of S defined by $f'(x) = 1 - f(x)$ for all $x \in S$. We have seen in

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[7] that a nonempty subset A of an ordered groupoid S is a left (resp. right) ideal of S if and only if its characteristic function f_A is a fuzzy left (resp. fuzzy right) ideal of S. A nonempty subset F of an ordered groupoid S is a filter of S if and only if the fuzzy subset f_F is a fuzzy filter of S. A fuzzy subset f of an ordered groupoid S is a fuzzy filter of S if and only if the complement f' of f is a fuzzy prime ideal of S. Later, fuzzy ordered semigroups have been widely studied by many authors.

In the present paper we examine the results of ordered groupoids given in [7] in case of some hypergroupoids. We deal with an hypergroupoid $(H, \circ)$ endowed with a relation "$\le$" (not order relation, and so not compatible with the hyperoperation "$\circ$" in general). Though we could call σ that relation and $\sigma-$hypergroupoid the hypergroupoid endowed with the relation σ, we will show by "$\le$" the relation and use the term $\le$–hypergroupoid, to emphasize the fact that our results hold for ordered hypergroupoids as well. As a consequence, the results in [7] also hold in groupoids endowed with a relation "$\le$" which is not an order in general and so no compatible with the multiplication in general. Our aim is to show the way we pass from ordered groupoids to ordered hypergroupoids.

For a groupoid $(S, \cdot)$ we have one operation corresponding to each $(a, b) \in S \times S$ the unique element ab of S. For an hypergroupoid H we have two "operations". One of them is the "operation" between the elements of H which is called "hyperoperation" as it maps the set $H \times H$ into the set of nonempty subsets of H and the other is the operation between the nonempty subsets of H. We use the terms left (right) ideal, bi-ideal, quasi-ideal instead of left (right) hyperideal, bi-hyperideal, quasi-hyperideal and so on, and this is because in this structure there are no two kind of left ideals, for example, to distinguish them as left ideal and left hyperideal. The left ideal in this structure is that one which corresponds to the left ideal of groupoids.

2 Main results

An *hypergroupoid* is a nonempty set H with an hyperoperation

$\circ : H \times H \to \mathcal{P}^*(H) \mid (a, b) \to a \circ b$ on H

and an operation

$* : \mathcal{P}^*(H) \times \mathcal{P}^*(H) \to \mathcal{P}^*(H) \mid (A, B) \to A * B$ on $\mathcal{P}^*(H)$

(induced by the operation of H) such that

$$A * B = \bigcup_{(a,b)\in A\times B} (a \circ b)$$

for every $A, B \in \mathcal{P}^*(H)$, $\mathcal{P}^*(H)$ being the set of (all) nonempty subsets of H. As the operation "$*$" depends on the hyperoperation "$\circ$", an hypergroupoid can be also denoted by $(H, \circ)$ (instead of $(H, \circ, *)$). If H is an hypergroupoid then, for every $x, y \in H$, we have $\{x\} * \{y\} = x \circ y$.

By the definition of the hypergroupoid we have the following proposition which, though clear, plays an essential role in the theory of hypersemigroups.

Proposition 1. [4, 5] *Let $(H, \circ)$ be an hypergroupoid, $x \in H$ and $A, B \in \mathcal{P}^*(H)$.*

Then we have the following:

(1) $x \in A * B \Longleftrightarrow x \in a \circ b$ *for some* $a \in A$, $b \in B$.

(2) *If* $a \in A$ *and* $b \in B$, *then* $a \circ b \subseteq A * B$.

It is well known that a nonempty subset A of a groupoid $(S, \cdot)$ is called a left (right) ideal of S if $SA \subseteq A$ (resp. $AS \subseteq A$). It is called a subgroupoid of S if $A^2 \subseteq A$ (cf., for example [1]). These concepts are naturally transferred in case of hypergroupoids as follows: A nonempty subset A of an hypergroupoid $(H, \circ)$ is called a *left* (resp. *right*) *ideal* of H if $H * A \subseteq A$ (resp. $A * H \subseteq A$). A subset of H which is both a left ideal and a right ideal of H is called an *ideal* of H. A nonempty subset A of H is called a *subgroupoid* of H if $A * A \subseteq A$. Clearly, every left ideal, right ideal or ideal of H is a subgroupoid of H.

Lemma 2. [5] *Let* $(H, \circ)$ *be an hypergroupoid. If A is a left (resp. right) ideal of H then, for every* $h \in H$ *and every* $a \in A$, *we have* $h \circ a \subseteq A$ *(resp.* $a \circ h \subseteq A$*). "Conversely", if A is a nonempty subset of H such that* $h \circ a \subseteq A$ *(resp.* $a \circ h \subseteq A$*) for every* $h \in H$ *and every* $a \in A$, *then the set A is a left (resp. right) ideal of H.*

Lemma 3. *Let* $(H, \circ)$ *be an hypergroupoid. If A is a subgroupoid of H then, for every* $a, b \in A$, *we have* $a \circ b \subseteq A$. *"Conversely", if A is a nonempty subset of H such that* $a \circ b \subseteq A$ *for every* $a, b \in A$, *then A is a subgroupoid of H.*

Definition 4. By a $\leq$–*hypergroupoid* we mean an hypergroupoid H endowed with a relation denoted by "$\leq$".
We write $b \geq a$ if $a \leq b$ (i.e. if (a, b) belongs to the relation $\leq$).

The concepts of fuzzy left (right) ideals of ordered groupoids introduced by Kehayopulu and Tsingelis in [7] are naturally transferred to $\leq$–hypergroupoids in the following definition:

Definition 5. (cf. also [5]) Let H be a $\leq$–hypergroupoid. A fuzzy subset f of H is called a *fuzzy left ideal* of H if

1. $x \leq y \Rightarrow f(x) \geq f(y)$ and
2. if $f(x \circ y) \geq f(y)$ for all $x, y \in H$, meaning that $x, y \in H$ and $u \in x \circ y$ implies $f(u) \geq f(y)$.

A fuzzy subset f of H is called a *fuzzy right ideal* of H if

1. $x \leq y \Rightarrow f(x) \geq f(y)$ and
2. if $f(x \circ y) \geq f(x)$ for all $x, y \in H$, in the sense that if $x, y \in H$ and $u \in x \circ y$, then $f(u) \geq f(x)$.

A fuzzy subset f of H is called a *fuzzy ideal* of H it is both a fuzzy left and a fuzzy right ideal of H. As one can easily see, a fuzzy subset f of H is a fuzzy ideal of H if and only if

1. $x \leq y$ implies $f(x) \geq f(y)$ and
2. if $f(x \circ y) \geq \max\{f(x), f(y)\}$ for all $x, y \in H$, in the sense that $x, y \in H$ and $u \in x \circ y$ implies $f(u) \geq \max\{f(x), f(y)\}$.

Following Zadeh, any mapping $f : H \to [0,1]$ of a $\leq$–hypergroupoid H into the closed interval $[0,1]$ of real numbers is called a *fuzzy subset* of H (or a *fuzzy set* in H) and f_A (: the characteristic function of A) is the mapping

$$f_A : H \to \{0,1\} \mid x \to f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

The concepts of left and right ideals of ordered groupoids introduced by Kehayopulu in [3] are as follows: If $(S, \cdot, \leq)$ is an ordered groupoid, a nonempty subset A of S is called a left (resp. right) ideal of S if (1) $SA \subseteq A$ (resp. $AS \subseteq A$) (that is, if A is a left (resp. right) ideal of the groupoid $(S, \cdot)$) and (2) if $a \in A$ and $S \ni b \leq a$, then $b \in A$. If A is both a left and right ideal of $(S, \cdot, \leq)$, then it is called an ideal of S. These concepts are naturally transferred in case of an $\leq$–hypergroupoid as follows:

Definition 6. [5] Let H be a $\leq$–hypergroupoid. A nonempty subset A of H is called a *left* (resp. *right*) *ideal* of H if
(1) $H * A \subseteq A$ (resp. $A * H \subseteq A$) and
(2) if $a \in A$ and $H \ni b \leq a$, then $b \in A$.

Proposition 7. (cf. also [5]) *Let H be a $\leq$–hypergroupoid. If L is a left ideal of H, then f_L is a fuzzy left ideal of H. "Conversely", if L is a nonempty subset of H such that f_L is a fuzzy left ideal of H, then L is a left ideal of H.*

Proof. $\Longrightarrow$. Let L be a left ideal of H. By definition, f_L is a fuzzy subset of H. Let $x \leq y$. If $y \notin L$, then $f_L(y) = 0$, so $f_L(x) \geq f_L(y)$. If $y \in L$, then $H \ni x \leq y \in L$ and, since L is a left ideal of H, we have $x \in L$. Then $f_L(x) = f_L(y) = 1$, so $f_L(x) \geq f_L(y)$. Let now $x, y \in H$ and $u \in x \circ y$. Then $f_L(u) \geq f_L(y)$. Indeed: If $y \in L$ then, by by Proposition 1, we have $x \circ y \subseteq H * L \subseteq L$, so $u \in L$, then $f_L(y) = f_L(u) = 1$, so $f_L(u) \geq f_L(y)$. If $y \notin L$, then $f_L(y) = 0 \leq f_L(u)$.
$\Longleftarrow$. Let $x \in H$ and $y \in L$. Then $x \circ y \subseteq L$. Indeed: Let $x \circ y \nsubseteq L$. Then there exists $u \in x \circ y$ such that $u \notin L$. Since $u \in x \circ y$, by hypothesis, we have $f_L(u) \geq f_L(y)$. Since $u \notin L$, we have $f_L(u) = 0$. Since $y \in L$, we have $f_L(y) = 1$, then $0 \geq 1$ which is impossible. Let now $x \in L$ and $H \ni y \leq x$. Then $y \in L$. Indeed: Since f_L is a fuzzy left ideal of H and $y \leq x$, we have $f_L(y) \geq f_L(x)$. Since $x \in L$, we have $f_L(x) = 1$. Then we have $f_L(y) \geq 1$. On the other hand, $f_L(y) \leq 1$, so we have $f_L(y) = 1$, then $y \in L$, and the proof is complete. □

In a similar way we prove the following:

Proposition 8. *Let H be a $\leq$–hypergroupoid. If R is a right ideal of H, then f_R is a fuzzy right ideal of H. "Conversely", if R is a nonempty subset of H such that f_R is a fuzzy right ideal of H, then R is a right ideal of H.*

Proposition 9. *If H is a $\leq$–hypergroupoid, a nonempty subset I of H is an ideal of H if and only if f_I is a fuzzy ideal of H.*

The concept of a filter of an ordered groupoid introduced by Kehayopulu in 1987 [2] is as follows: If $(S, \cdot, \leq)$ is an ordered groupoid, a nonempty subset F of S is called a filter of S if (1) if $a, b \in F$, then $ab \in F$. (2) if $a, b \in F$ such that

$ab \in F$, then $a \in F$ and $b \in F$. (3) if $a \in F$ and $S \ni b \geq a$, then $b \in F$ (that is, if F is a subgroupoid of the groupoid $(S, \cdot)$ satisfying the properties (2) and (3)). This concept is naturally transferred to ≤–hypergroupoids in Definition 10 below. It might be noted that the properties (1) and (2) of Definition 10 correspond to the properties (1) and (2) of filters of ordered groupoids but, in contrast to the case of ordered groupoids, they are not enough to prove basic results on ordered hypergroupoids. To overcome this difficulty, the property (3) in Definition 10 has been added. Our aim now is to characterize the filters of ≤–hypergroupoids in terms of fuzzy filters.

Definition 10. Let H be a ≤–hypergroupoid. A nonempty subset F of H is called a *filter* of H if

(1) if $x, y \in F$, then $x \circ y \subseteq F$.
(2) if $x, y \in H$ and $x \circ y \subseteq F$, then $x \in F$ and $y \in F$.
(3) if $x, y \in H$, then $x \circ y \subseteq F$ or $(x \circ y) \cap F = \emptyset$.
(4) if $x \in F$ and $H \ni y \geq x$, then $y \in F$.

So a filter of H is a subgroupoid of H satisfying the conditions (2)–(4).

Remark 11. Let H be a ≤–hypergroupoid, F a filter of H and $x, y \in H$. The following are equivalent:

(1) $x \circ y \subseteq F$ or $(x \circ y) \cap F = \emptyset$.
(2) if $x \notin F$ or $y \notin F$, then $(x \circ y) \cap F = \emptyset$.

Indeed: (1) $\Longrightarrow$ (2). Let $x \notin F$ or $y \notin F$. If $x \circ y \subseteq F$ then, since F is a filter of H, we have $x \in F$ and $y \in F$ which is impossible. Thus we have $x \circ y \nsubseteq F$. Then, by (1), we get $(x \circ y) \cap F = \emptyset$ and (2) is satisfied.
(2) $\Longrightarrow$ (1). Let $x \circ y \nsubseteq F$. If $x, y \in F$ then, since F is a filter of H, we have $x \circ y \subseteq F$ which is impossible. Thus we have $x \notin F$ or $y \notin F$. Then, by (2), we have $(x \circ y) \cap F = \emptyset$, and (1) holds true.

The concept of a fuzzy filter of an ordered groupoid introduced by Kehayopulu and Tsingelis in [7] is naturally transferred to a ≤–hypergroupoid in the following definition:

Definition 12. Let H be a ≤–hypergroupoid. A fuzzy subset f of H is called a *fuzzy filter* of H if

1. if $x \leq y$ implies $f(x) \leq f(y)$ and
2. if $f(x \circ y) = \min\{f(x), f(y)\}$ for every $x, y \in H$, in the sense that if $x, y \in H$ and $u \in x \circ y$, then $f(u) = \min\{f(x), f(y)\}$.

Proposition 13. *Let H be a ≤–hypergroupoid. If F is a filter of H, then the fuzzy subset f_F is a fuzzy filter of H. "Conversely", if F is a nonempty subset of H such that f_F is a fuzzy filter of H, then F is a filter of H.*

Proof. $\Longrightarrow$. Let $x \leq y$. If $x \notin F$, then $f_F(x) = 0$, so $f_F(x) \leq f_F(y)$. If $x \in F$, then $f_F(x) = 1$. Since $y \in H$ and $y \geq x \in F$, we have $y \in F$. Then $f_F(y) = 1$, and $f_F(x) \leq f_F(y)$. Let now $x, y \in H$ and $u \in x \circ y$. Then $f_F(u) = \min\{f_F(x), f_F(y)\}$. Indeed: (a) If $x \circ y \subseteq F$, then $x \in F$ and $y \in F$. Also $u \in F$. Then $f_F(x) = f_F(y) = f_F(u) = 1$, so $f_F(u) = \min\{f_F(x), f_F(y)\}$.

(b) Let $x \circ y \not\subseteq F$. Then $x \notin F$ or $y \notin F$ (since $x, y \in F$ implies $x \circ y \subseteq F$ which is impossible), then $f_F(x) = 0$ or $f_F(y) = 0$, and $\min\{f_F(x), f_F(y)\} = 0$. On the other hand, since $x \circ y \not\subseteq F$, we have $(x \circ y) \cap F = \emptyset$. Since $u \in x \circ y$, we have $u \notin F$. Then $f_F(u) = 0$, so $f_F(u) = \min\{f_F(x), f_F(y)\}$.

$\Longleftarrow$. Let $x, y \in F$. Then $x \circ y \subseteq F$. Indeed: Let $u \in x \circ y$. By hypothesis, we have $f_F(u) = \min\{f_F(x), f_F(y)\}$. Since $x, y \in F$, we have $f_F(x) = f_F(y) = 1$. Then $f_F(u) = 1$, and $u \in F$, so F is a subgroupoid of H. Let $x, y \in H$ such that $x \circ y \subseteq F$. Then $x \in F$ and $y \in F$. Indeed: Let $x \notin F$ or $y \notin F$. Then $f_F(x) = 0$ or $f_F(y) = 0$, hence $\min\{f_F(x), f_F(y)\} = 0$. Since $x \circ y \in \mathcal{P}^*(H)$, the set $x \circ y$ is nonempty. Take an element $u \in x \circ y$. Since f_F is a fuzzy filter of H, we have $f_F(u) = \min\{f_F(x), f_F(y)\}$, so $f_F(u) = 0$. On the other hand, since $u \in x \circ y \subseteq F$, we have $f_F(u) = 1$. We get a contradiction. Let $x, y \in H$ such that $x \circ y \not\subseteq F$. Then $(x \circ y) \cap F = \emptyset$. Indeed: Let $u \in (x \circ y) \cap F$. Since $u \in x \circ y$, by hypothesis, we have $f_F(u) = \min\{f_F(x), f_F(y)\}$. If $x \notin F$, then $f_F(x) = 0$, thus $f_F(u) = 0$. On the other site, since $u \in F$, we have $f_F(u) = 1$. We get a contadiction, so we have $x \in F$. In a similar way we prove that $y \in F$ and, since F is a subgroupoid of H, we have $x \circ y \subseteq F$, which is impossible. Finally, let $x \in F$ and $H \ni y \geq x$. Since f_F is a fuzzy filter of H, we have $1 \geq f_F(y) \geq f_F(x) = 1$, then $f_F(y) = 1$, so $y \in F$. Thus F is a filter of H. □

In what follows, for a fuzzy subset f of an $\leq$–hypergroupoid H we introduce the concept of the complement f' of f (which again is analogous to that one defined for ordered groupoids in [6]) and we prove that a fuzzy subset f of a $\leq$–hypergroupoid H is a fuzzy filter of H if and only if the complement f' of H is a fuzzy prime ideal of H.

Definition 14. Let H be an hypergroupoid or $\leq$–hypergoupoid and f a fuzzy subset of H. The fuzzy subset

$$f' : S \to [0, 1] \text{ defined by } f'(x) = 1 - f(x)$$

is called the *complement* of f in H.

We remark the following:
(a) If $x \in H$, then $(f')'(x) = 1 - f'(x) = f(x)$. Thus we have $f'' := (f')' = f$.
(b) $f(x) \leq f(y) \Longleftrightarrow f'(x) \geq f'(y)$ $(x, y \in H)$.
(c) $f(x) = f(y) \Longleftrightarrow f'(x) = f'(y)$ $(x, y \in H)$.

Lemma 15. (cf. also [6]) *Let H be an hypergroupoid, f a fuzzy subset of H and $x, y \in H$. Then we have*

$$1 - \min\{f(x), f(y)\} = \max\{f'(x), f'(y)\} \qquad (*)$$

Proof. Let $f(x) \leq f(y)$. Then $\min\{f(x), f(y)\} = f(x)$, thus $1 - \min\{f(x), f(y)\} = 1 - f(x) = f'(x)$. On the other hand, $f(x) \leq f(y)$ implies $f'(x) \geq f'(y)$, so we have $\max\{f'(x), f'(y)\} = f'(x)$ and $(*)$ is satisfied. If $f(y) \leq f(x)$, by symmetry, the relation $(*)$ also holds. □

Lemma 16. *Let H be an hypergroupoid, f a fuzzy subset of H and $x, y \in H$. The following are equivalent:*

(1) $f(x \circ y) = \min\{f(x), f(y)\}$.
(2) $f'(x \circ y) = \max\{f'(x), f'(y)\}$.

Proof. (1) $\Longrightarrow$ (2). Let $u \in x \circ y$. By (1), we have $f(u) = \min\{f(x), f(y)\}$. Then, by Lemma 15, we have

$$f'(u) = 1 - f(u) = 1 - \min\{f(x), f(y)\} = \max\{f'(x), f'(y)\},$$

and (2) holds true.
(2) $\Longrightarrow$ (1). Let $u \in x \circ y$. By (2) and Lemma 15, we have

$$f'(u) = \max\{f'(x), f'(y)\} = 1 - \min\{f(x), f(y)\}.$$

Then $f(u) = 1 - f'(u) = \min\{f(x), f(y)\}$, and (1) is satisfied. □

The concept of fuzzy prime subsets of groupoids or of ordered groupoids [7] is naturally transferred to hypergroupoids and to ordered hypergroupoids in the following definition.

Definition 17. [5] A fuzzy subset f of a groupoid (or a $\leq$–hypergroupoid) H is called a *fuzzy prime subset* of H if $f(x \circ y) \leq \max\{f(x), f(y)\}$ for all $x, y \in H$. That is, if $x, y \in H$ and $u \in x \circ y$, then $f(u) \leq \max\{f(x), f(y)\}$.

If H is a $\leq$–hypergroupoid, by a *fuzzy prime ideal* of H we clearly mean a fuzzy prime subset of H which is at the same time a fuzzy ideal of S. So a fuzzy subset of H is a fuzzy prime ideal of H if and only if the following assertions are satisfied:

1. $x \leq y \Rightarrow f(x) \geq f(y)$ and
2. if $f(x \circ y) = \max\{f(x), f(y)\}$ for all $x, y \in H$, in the sense that if $x, y \in H$ and $u \in x \circ y$, then $f(u) = \max\{f(x), f(y)\}$.

Proposition 18. *Let H be a $\leq$–hypergroupoid and f a fuzzy subset of H. Then f is a fuzzy filter of H if and only if the complement f' of f is a fuzzy prime ideal of H.*

Proof. $\Longrightarrow$. Let $x \leq y$. Since f is a fuzzy filter of H, we have $f(x) \leq f(y)$, then $f'(x) \geq f'(y)$. Let now $x, y \in H$. Since f is a fuzzy filter of H, we have $f(x \circ y) = \min\{f(x), f(y)\}$. Then, by Lemma 16, $f'(x \circ y) = \max\{f'(x), f'(y)\}$, thus f' is a fuzzy prime ideal of H.
$\Longleftarrow$. Let $x \leq y$. Since f' is a fuzzy ideal of H, we have $f'(x) \geq f'(y)$. Then $f(x) = f''(x) \leq f''(y) = f(y)$. Let now $x, y \in H$. Since f' is a fuzzy prime ideal of H, we have $f'(x \circ y) = \max\{f'(x), f'(y)\}$ then, by Lemma 16, $f(x \circ y) = \min\{f(x), f(y)\}$, thus f is a fuzzy filter of H. □

Let us finish with an example.

Example. We consider the $\leq$–hypergroupoid $H = \{a, b, c, d, e\}$ defined by the hyperoperation given in the table and the relation "$\leq$" below.

$\circ$	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a, b, d\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a, b, d\}$	$\{a, b\}$
c	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a, b, d\}$	$\{a, b, c\}$
d	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a, b, d\}$	$\{a, b, c\}$
e	$\{a\}$	$\{a, b\}$	$\{a, b, c\}$	$\{a, b, d\}$	$\{a, b, e\}$

$$\leq := \{(a,a),(a,b),(b,b),(b,d)\}.$$

One can easily check that the set $A = \{a,b,d\}$ is a left ideal of $(H,\circ,\leq)$ and that the characteristic function f_A of A, that is, the mapping

$$f_A : H \to \{0,1\} \mid x \to f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

is a fuzzy left ideal of H (the latest being also a consequence of Proposition 7).

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References

[1] A. H. Clifford, G. B. Preston, *The Algebraic Theory of Semigroups*, Amer. Math. Soc. Math. Surveys 7, Providence, Rhode Island 1961.

[2] N. Kehayopulu, *On weakly commutative poe-semigroups*, Semigroup Forum **34**, no. 3 (1987), 367–370.

[3] N. Kehayopulu, *On weakly prime ideals of ordered semigroups*, Math. Japon. **35**, no. 6 (1990), 1051–1056.

[4] N. Kehayopulu, *On hypersemigroups*, Pure Math. Appl. (PU.M.A.) **25**, no. 2 (2015), 151–156.

[5] N. Kehayopulu, *On fuzzy prime and fuzzy semiprime ideals of ≤–hypergroupoids*, J. Hyperstruct. **5**, (no. 2) (2016), 108–114.

[6] N. Kehayopulu, M. Tsingelis, *A note of fuzzy sets in semigroups*, Sci. Math. **2**, no. 3 (1999), 411–413 (electronic).

[7] N. Kehayopulu, M. Tsingelis, *Fuzzy sets in ordered groupoids*, Semigroup Forum **65**, no. 1 (2002), 128–132.

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