B-ALGEBRAS ACTING ON SETS

JOEMAR C. ENDAM[1] AND EMELYN C. BANAGUA[2]

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ABSTRACT. In this paper, we introduce the notion of a B-action of a B-algebra X on a set S. We show that a B-action $*'$ of X on S induces an equivalence relation on S defined by $s \sim s'$ if and only if $x *' s = s'$ for some $x \in X$. Moreover, for any $s \in S$, the cardinality of the equivalence class $[s]_B$ of s is equal to the index of the corresponding subalgebra X_s in X, that is, $|[s]_B| = [X : X_s]_B$, where $X_s = \{x \in X : x *' s = s\}$. Furthermore, the number of distinct equivalence classes is given by $\frac{1}{|X|} \sum_{x \in X} F(x)$, where $F(x)$ is the number of elements of S fixed by x. We also introduce B-faithfulness and B-transitivity and investigate some related properties.

1 Introduction and Preliminaries In [3], the notion of B-algebras was introduced by J. Neggers and H.S. Kim in 2002. A *B-algebra* is an algebra $(X; *, 0)$ of type (2, 0) (that is, a nonempty set X with a binary operation $*$ and a constant 0) satisfying the following axioms for all $x, y, z \in X$:
(I) $x * x = 0$,
(II) $x * 0 = x$,
(III) $(x * y) * z = x * (z * (0 * y))$.
A B-algebra $(X; *, 0)$ is *commutative* [3] if $x * (0 * y) = y * (0 * x)$ for all $x, y \in X$. In [4], J. Neggers and H.S. Kim introduced the notions of a subalgebra and normality of B-algebras and some of their properties are established. A nonempty subset N of X is called a *subalgebra* of X if $x * y \in N$ for any x, $y \in N$. It is called *normal* in X if for any $x, y, a, b \in N$ ($x * y, a * b \in N$ implies $(x * a) * (y * b) \in N$). A normal subset of X is a subalgebra of X. There are several properties of B-algebras as established by some authors. The following properties are used in this paper, for any $x, y, z \in X$, we have
(P1) $0 * (0 * x) = x$ [3],
(P2) $x * y = 0 * (y * x)$ [5],
(P3) $x * (y * z) = (x * (0 * z)) * y$ [3],
(P4) $0 * x = 0 * y$ implies $x = y$ [3],
(P5) $x * y = 0$ implies $x = y$ [3].

In [1], the concept of B-cosets of B-algebras is introduced. Let H be a subalgebra of a B-algebra X and $x \in X$. Let $xH = \{x * (0 * h) : h \in H\}$ and $Hx = \{h * (0 * x) : h \in H\}$, called the *left* and *right B-cosets* of H in X, respectively. If X is commutative, then $xH = Hx$ for all $x \in X$. Observe that $0H = H = H0$ and $x = x * (0 * 0) \in xH$ and $x = 0 * (0 * x) \in Hx$. Also, $xH = H$ if and only if $x \in H$.

Theorem 1.1. *[1] Let H be a subalgebra of a B-algebra X and $a, b \in X$. Then*
i. $aH = bH$ if and only if $(0 * b) * (0 * a) \in H$
ii. $Ha = Hb$ if and only if $a * b \in H$.

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If H is a subalgebra of a B-algebra X, then $\{xH : x \in X\}$ forms a partition of X and there is a one-one correspondence of the set of all left B-cosets of H in X onto the set of all right B-cosets of H in X. Thus, we define the number of distinct left (or right) B-cosets, written $[X : H]_B$, of H in X as the *index* of H in X. If X is finite, then clearly $[X : H]_B$ is finite.

Theorem 1.2. *[1] (Lagrange's Theorem for B-algebras) Let H be a subalgebra of a finite B-algebra X. Then $|X| = [X : H]_B|H|$.*

In [2], the concepts of centralizer and normalizer of a B-algebra X are introduced. The *centralizer C(x) of x in X* is defined by $C(x) = \{y \in X : y*(0*x) = x*(0*y)\}$. Then $C(x)$ is a subalgebra of X for all $x \in X$. If H is a nonempty subset of X, then the *centralizer C(H) of H in X* is defined by $C(H) = \{y \in X : y*(0*x) = x*(0*y) \text{ for all } x \in H\}$. Since $C(H) = \bigcap_{x \in H} C(x)$, $C(H)$ is a subalgebra of X. In particular, the center $C(X) = Z(X)$ of X is a subalgebra of X. Now, let H and K be nonempty subsets of X. For every $x \in X$, we define H_x as the set $H_x = \{x * (x * h) : h \in H\}$. The *normalizer of H in K*, denoted by $N_K(H)$, is defined by $N_K(H) = \{x \in K : H_x = H\}$. If $K = X$, then $N_X(H)$ is called the *normalizer of H*, denoted by $N(H)$. If K is a subalgebra of X, then $N_K(H)$ is a subalgebra of X. In particular, $N(H)$ is a subalgebra of X.

2 B-algebras acting on sets

This section introduces the notion of a B-action on a set. It also provides some related properties.

Definition 2.1. A *B-action* of a B-algebra X on a set S is a map $*' : X \times S \to S$, written $x *' s$ for all $(x, s) \in X \times S$, satisfying the following properties:
(B1) $0 *' s = s$
(B2) $x_1 *' (x_2 *' s) = (x_1 * (0 * x_2)) *' s$.
When such a B-action is given, we say that X acts on the set S.

Example 2.2. Let X be a B-algebra and S be a nonempty set. Define $*' : X \times S \to S$ by $(x, s) \to s$ for all $(x, s) \in X \times S$. Clearly, $0 *' s = s$. Let $x_1, x_2 \in X$ and $s \in S$. Then $x_1 *' (x_2 *' s) = x_1 *' s = s = (x_1 * (0 * x_2)) *' s$. Thus, $*'$ is a B-action and is called the *trivial B-action* of X on S.

Example 2.3. Let X be a B-algebra and H be a subalgebra of X. Define $*' : H \times X \to X$ by $(h, x) \to h * (0 * x)$ for all $(h, x) \in H \times X$. Let $h_1, h_2 \in H$ and $x \in X$. Then by (P2), (P1), and (III), we have

$$\begin{aligned} h_1 *' (h_2 *' x) &= h_1 *' (h_2 * (0 * x)) \\ &= h_1 * [0 * (h_2 * (0 * x))] \\ &= h_1 * [(0 * x) * h_2] \\ &= h_1 * [(0 * x) * (0 * (0 * h_2))] \\ &= (h_1 * (0 * h_2)) * (0 * x) \\ &= (h_1 * (0 * h_2)) *' x. \end{aligned}$$

By (P1), $0 *' x = 0 * (0 * x) = x$. Thus, $*'$ is a B-action and is called *left B-translation* of H on X.

Example 2.4. Let X be a B-algebra and H, K be subalgebras of X. Let $\mathcal{L}$ be the set of all left B-cosets of K in X. Define $*' : H \times \mathcal{L} \to \mathcal{L}$ by $(h, xK) \to (h * (0 * x))K$. Then H acts on $\mathcal{L}$ by left B-translation.

Example 2.5. Let X be a B-algebra and H be a subalgebra of X. Define $*' : H \times X \to X$ by $(h, x) \to h * (h * x)$ for all $(h, x) \in H \times X$. Let $h_1, h_2 \in H$ and $x \in X$. Then by (P3) and (P2), we have

$$\begin{aligned} h_1 *' (h_2 *' x) &= h_1 *' (h_2 * (h_2 * x)) \\ &= h_1 * (h_1 * (h_2 * (h_2 * x))) \\ &= h_1 * (h_1 * [(h_2 * (0 * x)) * h_2]) \\ &= h_1 * [(h_1 * (0 * h_2)) * (h_2 * (0 * x))] \\ &= [h_1 * (0 * (h_2 * (0 * x)))] * (h_1 * (0 * h_2)) \\ &= [h_1 * ((0 * x) * h_2)] * (h_1 * (0 * h_2)) \\ &= [(h_1 * (0 * h_2)) * (0 * x)] * (h_1 * (0 * h_2)) \\ &= (h_1 * (0 * h_2)) * [(h_1 * (0 * h_2)) * x] \\ &= (h_1 * (0 * h_2)) *' x. \end{aligned}$$

By (P1), $0 *' x = 0 * (0 * x) = x$. Thus, $*'$ is a B-action and is called *B-conjugation*.

Lemma 2.6. *Let X be a B-algebra and $*'$ be a B-action of X on a set S. Let $s_1, s_2 \in S$ and $x \in X$. If $x *' s_1 = x *' s_2$, then $s_1 = s_2$.*

Proof. Let $s_1, s_2 \in S$ and $x \in X$. Suppose that $x *' s_1 = x *' s_2$. Then by (B2), (I), and (B1), we have

$$\begin{aligned} x *' s_1 &= x *' s_2 \\ (0 * x) *' (x *' s_1) &= (0 * x) *' (x *' s_2) \\ ((0 * x) * (0 * x)) *' s_1 &= ((0 * x) * (0 * x)) *' s_2 \\ 0 *' s_1 &= 0 *' s_2 \\ s_1 &= s_2 \end{aligned}$$

This proves the lemma. □

Lemma 2.7. *Let X be a B-algebra and $*'$ be a B-action of X on a set S. Let $x \in X$, $s, r \in S$. Then $x *' s = r$ if and only if $s = (0 * x) *' r$.*

Proof. If $x *' s = r$, then by (B2), (I), and (B1), we obtain

$$\begin{aligned} (0 * x) *' (x *' s) &= (0 * x) *' r \\ ((0 * x) * (0 * x)) *' s &= (0 * x) *' r \\ 0 *' s &= (0 * x) *' r \\ s &= (0 * x) *' r \end{aligned}$$

Conversely, if $s = (0 * x) *' r$, then by (B2), (P1), (I), and (B1), we obtain

$$\begin{aligned} s &= (0 * x) *' r \\ x *' s &= x *' ((0 * x) *' r) \\ x *' s &= (x * (0 * (0 * x))) *' r \\ x *' s &= (x * x) *' r \\ x *' s &= 0 *' r \\ x *' s &= r \end{aligned}$$

This completes the proof. □

Theorem 2.8. *Let X be a B-algebra and $*'$ be a B-action of X on a set S. Define $\sim$ on S by $s \sim s'$ if and only if $x *' s = s'$ for some $x \in X$. Then $\sim$ is an equivalence relation on S.*

Proof. Let $s \in S$. By (B1), $0 *' s = s$ and so $s \sim s$. Hence, $\sim$ is reflexive. Now, let $s, s' \in S$. Suppose that $s \sim s'$, then there exists $x \in X$ such that $x*'s = s'$. Note that $0*x \in X$ and by (B2), (I), and (B1), we have $(0*x)*'s' = (0*x)*'(x*'s) = ((0*x)*(0*x))*'s = 0*'s = s$. Hence, $s' \sim s$ and so $\sim$ is symmetric. Let $s_1, s_2, s_3 \in S$. Suppose that $s_1 \sim s_2$ and $s_2 \sim s_3$. Then there exist $x_1, x_2 \in X$ such that $x_1 *' s_1 = s_2$ and $x_2 *' s_2 = s_3$. Note that $x_2 * (0 * x_1) \in X$ and by (B2), we have $(x_2 * (0 * x_1)) *' s_1 = x_2 *' (x_1 * s_1) = x_2 *' s_2 = s_3$. Thus, $s_1 \sim s_3$ and so $\sim$ is transitive. Therefore, $\sim$ is an equivalence relation on S. □

Theorem 2.9. *Let X be a B-algebra and $*'$ be a B-action of X on a set S. Then for each $s \in S$, $X_s = \{x \in X : x *' s = s\}$ is a subalgebra of X.*

Proof. Let $s \in S$. By (B1) , $0 \in X_s$ and so $X_s \neq \varnothing$. Let $a, b \in X_s$. Then $a, b \in X$ such that $a *' s = s$ and $b *' s = s$. Now, by (B2), (I), and (B1), we have $(0 * b) *' s = (0 * b) *' (b *' s) = ((0 * b) * (0 * b)) *' s = 0 *' s = s$. Thus, by (B2) and (P1), we have $s = a *' s = a *' ((0 * b) *' s) = (a * (0 * (0 * b))) *' s = (a * b) *' s$. Hence, $a * b \in X_s$ and so X_s is a subalgebra of X. □

The equivalence classes of the equivalence relation of Theorem 2.8 are called *B-orbits of X on S* and the B-orbit of $s \in S$ is denoted by $[s]_B$. The subalgebra X_s in Theorem 2.9 is called *B-stabilizer of s*.

Example 2.10. Let X be a B-algebra and $*'$ be a left B-translation of X on itself. Then there is only one B-orbit of X. To see this, $[0]_B = \{y \in X : x *' 0 = y \text{ for some } x \in X\} = \{y \in X : x * (0 * 0) = y \text{ for some } x \in X\} = \{y \in X : x = y \text{ for some } x \in X\} = X$. For any $x \in X$, the B-stabilizer of x is trivial. To see this, $X_x = \{y \in X : y *' x = x\} = \{y \in X : y * (0 * x) = x\} = \{y \in X : y = 0\} = \{0\}$.

Example 2.11. Let X be a B-algebra and $*'$ be a B-conjugation of X on itself. For any $x \in X$, the B-orbit of x is the conjugacy class of x and the B-stabilizer of x is the centralizer of x.

The following theorem tells us that the cardinality of the B-orbit $[s]_B$ of s is equal to the index of the B-stabilizer X_s in X.

Theorem 2.12. *Let X be a B-algebra and $*'$ be a B-action of X on S. For $s \in S$, we have $|[s]_B| = [X : X_s]_B$.*

Proof. Let $s \in S$. Let $\mathcal{L}$ be the collection of all left B-cosets of X_s in X. Let $r \in [s]_B$. Then there exists $x_r \in X$ such that $x_r *' s = r$. Define $\varphi : [s]_B \to \mathcal{L}$ by $\varphi(r) = x_r X_s$.
Claim 1: φ is well-defined.
Clearly, $\varphi(r) = x_r X_s \in \mathcal{L}$ for all $r \in [s]_B$. Let $p, q \in [s]_B$ such that $p = q$. Then there exist $x_p, x_q \in X$ such that $x_p *' s = p$ and $x_q *' s = q$. Hence, $x_p *' s = x_q *' s$. Now, by (B2), (I), and (B1), we have

$$\begin{aligned} x_p *' s = x_q *' s &\Rightarrow (0 * x_q) *' (x_p *' s) = (0 * x_q) *' (x_q *' s) \\ &\Rightarrow ((0 * x_q) * (0 * x_p)) *' s = ((0 * x_q) * (0 * x_q)) *' s \\ &\Rightarrow ((0 * x_q) * (0 * x_p)) *' s = ((0 *' s) \\ &\Rightarrow ((0 * x_q) * (0 * x_p)) *' s = s \end{aligned}$$

Thus, $((0 * x_q) * (0 * x_p)) \in X_s$. By Theorem 1.1(i), $\varphi(p) = x_p X_s = x_q X_s = \varphi(q)$ and so φ is well-defined.

Claim 2: φ is one-to-one.
Suppose that $p, q \in [s]_B$ and $\varphi(p) = \varphi(q)$. Then there exist $x_q, x_p \in X$ such that $p = x_p *' s$, $q = x_q *' s$ and $x_pX_s = x_qX_s$. By Theorem 1.1(i), $(0 * x_q) * (0 * x_p) \in X_s$. Then by (B1), (I), and (B2), we have

$$\begin{aligned}((0 * x_q) * (0 * x_p)) *' s &= s \\ ((0 * x_q) * (0 * x_p)) *' s &= (0 *' s) \\ ((0 * x_q) * (0 * x_p)) *' s &= ((0 * x_q) * (0 * x_q)) *' s \\ (0 * x_q) *' (x_p *' s) &= (0 * x_q) *' (x_q *' s)\end{aligned}$$

By Lemma 2.6, $x_p *' s = x_q *' s$ and so $p = q$. Thus, φ is one-to-one.
Claim 3: φ is onto.
If $x_rX_s \in \mathcal{L}$, then $\varphi(r) = \varphi(x_r *' s) = x_rX_s$. Hence, φ is onto.
Therefore, φ is bijective. Consequently, $|[s]_B| = [X : X_s]_B$. □

Corollary 2.13. *Let X be a finite B-algebra that acts on a set S and $s \in S$. Then $|X| = |[s]_B||X_s|$.*

Proof. This follows from Theorems 1.2 and 2.12. □

Corollary 2.14. *Let X be a finite B-algebra that acts on a set S. If S is finite, then $|S| = \sum_{a\in A}[X : X_a]_B$, where A is a subset of S containing exactly one element from each B-orbit $[a]_B$.*

Proof. This follows from Theorems 2.8 and 2.12. □

Definition 2.15. Let X be a B-algebra and $*'$ be a B-action of X on a set S. Let $s \in S$ and $x \in X$. Then s is called *fixed* by x if $x *' s = s$. If $x *' s = s$ for all $x \in X$, then s is called fixed by X. We also define $F(x)$ as the number of elements of S fixed by x.

Theorem 2.16. *Let X be a B-algebra and $*'$ be a B-action of X on a nonempty finite set S. Then the number of B-orbits of X is given by $\dfrac{1}{|X|}\sum_{x\in X} F(x)$.*

Proof. Let $T = \{(x, s) \in X \times S : x*'s = s\}$. Since $F(x)$ is the number of elements $s \in S$ such that $(x, s) \in T$, it follows that $|T| = \sum_{x\in X} F(x)$. Also, $|X_s|$ is the number of elements $x \in X$ such that $(x, s) \in T$. Hence, $|T| = \sum_{s\in S}|X_s|$. Let $S = [s_1]_B \cup [s_2]_B \cup \cdots \cup [s_k]_B$, where $\{[s_1]_B, [s_2]_B, \ldots, [s_k]_B\}$ is the set of all distinct B-orbits of X on S. Then $\sum_{x\in X} F(x) = \sum_{s\in[s_1]_B}|X_s| + \sum_{s\in[s_2]_B}|X_s| + \cdots + \sum_{s\in[s_k]_B}|X_s|$. Suppose that a and b are in the same B-orbit. Then $[a]_B = [b]_B$ and so by Theorem 2.12, $[X : X_a] = |[a]_B| = |[b]_B| = [X : X_b]$. By Theorem 1.2, $\dfrac{|X|}{|X_a|} = \dfrac{|X|}{|X_b|}$ and so $|X_a| = |X_b|$. Thus, $\sum_{x\in X} F(x) = |[s_1]_B||X_{s_1}| + |[s_2]_B||X_{s_2}| + \cdots + |[s_k]_B||X_{s_k}| = \dfrac{|X|}{|X_{s_1}|}|X_{s_1}| + \dfrac{|X|}{|X_{s_2}|}|X_{s_2}| + \cdots + \dfrac{|X|}{|X_{s_k}|}|X_{s_k}| = k|X|$, where k is the number of distinct B-orbits. Consequently, $k = \dfrac{1}{|X|}\sum_{x\in X} F(x)$. □

Let X be a B-algebra and $*'$ be a B-action of X on a set S. For the succeeding results, let $S_0 = \{s \in S : x *' s = s \text{ for all } x \in X\}$.

Theorem 2.17. *Let X be a B-algebra and $*'$ be a B-action of X on a finite set S. If $|X| = p^n$ for some prime p, then $|S| \equiv |S_0| \mod p$.*

Proof. By Corollary 2.14, $|S| = \sum_{a\in A}[X : X_a]_B$, where A is a subset of S containing exactly one element from each B-orbit $[a]_B$. Now, $s \in S_0$ if and only if $x *' s = s$ for all $x \in X$ if and only if $[s]_B = \{s\}$. Hence, $|S| = |S_0| + \sum_{a\in A\setminus S_0} \frac{|X|}{|X_a|}$. Since $|X_a| \neq |X|$ for all $a \in A \setminus S_0$, $\frac{|X|}{|X_a|}$ is some power of p for all $a \in A \setminus S_0$. Thus, $\frac{|X|}{|X_a|}$ is divisible by p. Therefore, $|S| \equiv |S_0| \mod p$. □

Corollary 2.18. *Let X be a B-algebra and $*'$ be a B-action of X on a finite set S. If $|X| = p^n$ for some prime p such that p does not divide $|S|$, then there exists $s \in S$ such that s is fixed by X.*

Proof. By Theorem 2.17, $|S| \equiv |S_0| \mod p$. Since p does not divide $|S|$, p does not divide $|S_0|$. Thus, $|S_0| \neq 0$. Hence, there exists $s \in S_0$. Therefore, s is fixed by X. □

Theorem 2.19. *Let H be a subalgebra of a finite B-algebra X, where $|H| = p^k$ for some prime p and nonnegative integer k. Then $[X : H]_B \equiv [N(H) : H]_B \mod p$. Moreover, if p divides $[X : H]_B$, then $N(H) \neq H$.*

Proof. Let $\mathcal{L} = \{xH : x \in X\}$. Define $*' : H \times \mathcal{L} \to \mathcal{L}$ by $(h, xH) \to (h * (0 * x))H$. Then $*'$ is a left B-translation of H on $\mathcal{L}$. Let $\mathcal{L}_0 = \{xH \in \mathcal{L} : h *' xH = xH \text{ for all } h \in H\}$. By Theorem 2.17, $|\mathcal{L}| \equiv |\mathcal{L}_0| \mod p$. Now, $xH \in \mathcal{L}_0$ if and only if $h *' xH = xH$ for all $h \in H$ if and only if $\{x * (x * h) : h \in H\} = H$ if and ony if $x \in N(H)$. Thus, $\mathcal{L}_0$ is the set of all left B-cosets of H in $N(H)$. Hence, $|\mathcal{L}_0| = [N(H) : H]_B$. Also, $|\mathcal{L}| = [X : H]_B$. Therefore, $[X : H]_B \equiv [N(H) : H]_B \mod p$. Moreover, if p divides $[X : H]_B$, then p divides $[N(H) : H]_B$. Since $[N(H) : H]_B \geq 1$, it follows that $N(H) \neq H$. □

3 B-faithful and B-transitive This section presents two kinds of B-actions on a set. These B-actions are B-faithful and B-transitive.

Definition 3.1. A B-action of a B-algebra X on a set S is called *B-faithful* if $x *' s = s$ for all $s \in S$ implies that $x = 0$. A B-action of X on S is called *B-transitive* if for all $s, r \in S$ there exists $x \in X$ such that $x *' s = r$.

Example 3.2. The B-action of a B-algebra X on itself by left B-translation is faithful.

Example 3.3. The B-action of a B-algebra X on itself by B-conjugation is faithful if and only if $Z(X) = \{0\}$.

Theorem 3.4. *Let X be a B-algebra and $*'$ be a B-action of X on a set S. Then $*'$ is B-faithful on S if and only if no two distinct elements of X have the same B-action on each element of S.*

Proof. Suppose that $x, y \in X$ such that $x *' s = y *' s$ for any $s \in S$. Then by (B2), (I), and (B1), we have

$$\begin{aligned} x *' s &= y *' s \\ (0 * y) *' (x *' s) &= (0 * y) *' (y *' s) \\ ((0 * y) * (0 * x)) *' s &= ((0 * y) * (0 * y)) *' s \\ ((0 * y) * (0 * x)) *' s &= 0 *' s \\ ((0 * y) * (0 * x)) *' s &= s \end{aligned}$$

Since $*'$ is B-faithful, $(0 * y) * (0 * x) = 0$. By (P5), $(0 * y) = (0 * x)$. Applying (P4), $y = x$. Conversely, let $x \in X$ such that $x *' s = s$ for any $s \in S$, then $x = 0$ since x has the same B-action on S as 0. Thus, the B-action is B-faithful. □

Theorem 3.5. *Let X be a B-algebra and $*'$ be a B-transitive of X on a set S containing at least two elements. Then for any $s \in S$, $[s]_B = S$ and $|S| = [X : X_s]_B$.*

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[1]Department of Mathematics
Negros Oriental State University
Kagawasan Ave., Dumaguete City, PHILIPPINES
jc endam@yahoo.com

[2]Department of Mathematics
Silliman University
Hibbard Ave., Dumaguete City, PHILIPPINES
emelyncbanagua@su.edu.ph