ON SUBWEAKLY b-CONTINUOUS FUNCTIONS

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Abstract. The purpose of this paper is to introduce a new class functions called, subweakly b-continuous functions. Also, we obtain its characterizations and its basic properties.

1 Introduction Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of continuous functions have been introduced over the years. Various interesting problems arise when one considers continuity. Its importance is significant in various areas of mathematics and related sciences. In 1996, Andrijevic [2] introduced a weak form of open sets called b-open sets. In the same year, this notion was also called sp-open sets in the sense of Dontchev and Przemski [6] but one year later are called γ-open sets due to El-Atik [14]. Ekici [3, 7, 8, 9, 10, 11, 12, 13] studied some papers related with b-open sets. In this paper, we will continue the study of related functions by involving b-open sets. We introduce and characterize the concept of subweakly b-continuous functions in topological spaces.

2 Preliminaries Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f : (X, \tau) \to (Y, \sigma)$ (or simply $f : X \to Y$) denotes a function f of a space (X, τ) into a space (Y, σ). Let A be a subset of a space (X, τ). The closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. The θ-closure [24] of A, denoted by $Cl_\theta(A)$, is defined to be the set of all $x \in X$ such that $A \cap Cl(U) \neq \emptyset$ for every open neighbourhood U of x. If $A = Cl_\theta(A)$, then A is called θ-closed. The complement of θ-closed set is called θ-open. A subset A of (X, τ) is said to be regular open [23](resp. semi-open [15], preopen [16], α-open [20], b-open [2] or γ-open [14]) if $A = Int(Cl(A))$ (resp. $A \subset Cl(Int(A)), A \subset Int(Cl(A)), A \subset Int(Cl(Int(A))), A \subset Int(Cl(A)) \cup Cl(Int(A))$). The complement of a semi-open (resp. preopen, b-open) set is called semi-closed [5] (resp.preclosed [16], b-closed [2]). The intersection of all semi-closed (resp. preclosed, b-closed) sets containing A is called the semiclosure [4] (resp. preclosure [16], b-closure [2]) of A and is denoted by $sCl(A)$ (resp. $pCl(A), bCl(A)$). For each $x \in X$, the family of all b-open sets containing x is denoted by $BO(X, \tau; x)$. The family of all α-open (resp. b-open) sets of a topological space (X, τ) is denoted by $\alpha O(X, \tau)$ (resp. $BO(X, \tau)$). A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *α-continuous* [17] if for every $x \in X$ and every open set V of Y containing $f(x)$, there exists an α-open set U containing x such that $f(U) \subset V$. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *weakly b-continuous* [22] if for every $x \in X$ and every open set V of Y containing $f(x)$, there exists $U \in BO(X, \tau; x)$ such that $f(U) \subset Cl(V)$.

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Lemma 2.1 [2] *Let A be a subset of a topological space (X, τ).*
(i) $x \in bCl(A)$ *if and only if* $A \cap U \neq \emptyset$ *for every* $U \in BO(X, \tau; x)$.
(ii) *Any union of b-open sets is b-open.*
(iii) $bCl(A)$ *is b-closed.*
(iv) A *is b-closed if and only if* $A = bCl(A)$.

3 Subweakly b-continuous functions

Definition 3.1 A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *subweakly b-continuous* if there exists an open base $\mathcal{B}$ for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$.

Remark 3.2 (i) It is clear that weak b-continuity implies subweak b-continuity.
(ii) The converse of the implication of (i) above is not true in general as it can be seen from the following example: let (X, τ) and (Y, σ) be the following topological spaces, where $X := \{a, b, c, d\} = Y$, $\tau := \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma := P(Y)$ and $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = f(b) := a$, $f(c) := b$, $f(d) := c$. Then, there exists an open base $\mathcal{B}$ of the topology σ on Y such that $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every set $V \in \mathcal{B}$,i.e., $f : (X, \tau) \to (Y, \sigma)$ is subweakly b-continuous. Indeed, we take $\mathcal{B} := \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, Y\}$ and we have that: $BO(X, \tau) = P(X) \setminus \{\{d\}\}$ and $BC(X, \tau) = P(X) \setminus \{\{a, b, c\}\}$. And, the function $f : (X, \tau) \to (Y, \sigma)$ is not weakly b-continuous. Indeed. there exist a point $d \in X$ and a set $V := \{c, d\} \in \sigma$ such that $f(d) = c \in V$ and $f(U) \not\subset Cl(V)$ for every set $U \in BO(X, \tau; d)$, where $BO(X, \tau; d)= \{\{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$.

Theorem 3.3 *A function $f : (X, \tau) \to (Y, \sigma)$ is subweakly b-continuous if and only if there is an open base $\mathcal{B}$ for the topology σ on Y for which $Cl(Int(f^{-1}(V))) \cap Int(Cl(f^{-1}(V))) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$.*

Proof. The proof is clear, because it is well known that $bCl(A) = A \cup (Cl(Int(A)) \cap Int(Cl(A)))$ holds for every set A of (X, τ) (cf. Proposition 2.5 and its proof in [2]). **q.e.d**

Theorem 3.4 *If a function $f : (X, \tau) \to (Y, \sigma)$ is subweakly b-continuous, then for every θ-open* (resp. *θ-closed*) *set V of (Y, σ), $f^{-1}(V)$ is a union of b-closed sets* (resp. *an intersection of b-open sets*).

Proof. Let $\mathcal{B}$ be an open base for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Let V be a θ-open subset of (Y, σ) with $x \in f^{-1}(V)$. Then there exist $W \in \mathcal{B}$ such that $f(x) \in W \subset Cl(W) \subset V$. Then $x \in bCl(f^{-1}(W)) \subset f^{-1}(Cl(W)) \subset f^{-1}(V)$. By Lemma 2.1, $bCl(f^{-1}(W))$ is b-closed; $f^{-1}(V)$ is a union of b-closed sets. The second case is proved by an argument similar to the first case above. **q.e.d.**

Recall that by the graph of a function $f : X \to Y$, we mean that $G(f) := \{(x, y)| \ x \in X, y = f(x)\}$ and by the graph function of f, say $g : X \to Y$, we mean that $g(x) := (x, f(x))$.

Theorem 3.5 *If $f : (X, \tau) \to (Y, \sigma)$ is subweakly b-continuous and (Y, σ) is Hausdorff, then $G(f)$ is b-closed in $(X \times Y, \tau \times \sigma)$.*

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Let $\mathcal{B}$ be an open base for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Since (Y, σ)

is Hausdorff, there exist disjoint open sets V and W in Y with $y \in V$, $f(x) \in W$, and $V \in \mathcal{B}$. Then $f(x) \notin Cl(V)$ and hence $x \notin f^{-1}(Cl(V))$. Since f is subweakly b-continuous, $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ and hence $x \notin bCl(f^{-1}(V))$. Then we see that $(x,y) \in (X \setminus bCl(f^{-1}(V))) \times V \subset (X \times Y) \setminus G(f)$. Then by Lemmas 2.1, we have that $G(f)$ is b-closed. **q.e.d**

Theorem 3.6 *If $f : (X,\tau) \to (Y,\sigma)$ is subweakly b-continuous, then the graph function $g : (X,\tau) \to (X \times Y, \tau \times \sigma)$ is subweakly b-continuous.*

Proof. Let $\mathcal{B}$ be an open base for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Then $\mathcal{C} := \{U \times V | U \subset X$ is open and $V \in \mathcal{B}\}$ is an open base for the product topology $\tau\times\sigma$ on $X\times Y$. For $U\times V \in \mathcal{C}$, we have $bCl(g^{-1}(U\times V)) = bCl(U \cap f^{-1}(V)) \subset bCl(U) \cap bCl(f^{-1}(V)) \subset Cl(U) \cap f^{-1}(Cl(V)) = g^{-1}(Cl(U)\times Cl(V)) = g^{-1}(Cl(U \times V))$. Thus, the graph function $g : (X,\tau) \to (X \times Y, \tau \times \sigma)$ is subweakly b-continuous. **q.e.d.**

Theorem 3.7 *Let $f : (X,\tau) \to (Y,\sigma)$ be a function and $g : (X,\tau) \to (X \times Y, \tau \times \sigma)$ its graph function. Let $\mathcal{B}$ be an open base for the topology σ on Y. If $g : (X,\tau) \to (X\times Y, \tau\times\sigma)$ is subweakly b-continuous with respect to the open base $\mathcal{C} = \{U \times V |\ U \subset X$ is open and $V \in \mathcal{B}\}$ for the product topology $\tau \times \sigma$ on $X \times Y$, then f is subweakly b-continuous with respect to the open base $\mathcal{B}$.*

Proof. Let $V \in \mathcal{B}$. We have $bCl(f^{-1}(V)) = bCl(X \setminus f^{-1}(V)) = bCl(g^{-1}(X \times V)) \subset g^{-1}(Cl(X \times V)) = g^{-1}(X \times Cl(V))) = f^{-1}(Cl(V))$; hence f is subweakly b-continuous. **q.e.d.**

Definition 3.8 A topological space (X,τ) is said to be b-T_1 [10] if for each pair of distinct points x and y of X, there exists b-open sets U and V containing x and y, respectively such that $y \notin U$ and $x \notin V$.

Theorem 3.9 *If $f : (X,\tau) \to (Y,\sigma)$ is subweakly b-continuous injection and (Y,σ) is Hausdorff, then (X,τ) is b-T_1.*

Proof. Let x and y be distinct points in X. Since f is injective, $f(x) \neq f(y)$. Let $\mathcal{B}$ be an open base for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Since (Y,σ) is Hausdorff, there exist disjoint subsets V_1 and W_1 in (Y,σ) with $f(y) \in V_1$, $f(x) \in W_1$. There exists a subset $V \in \mathcal{B}$ such that $f(y) \in V, f(x) \notin V$, $V \cap W_1 = \emptyset$ and $V \subset V_1$. Then $f(x) \notin Cl(V)$; and hence $y \in f^{-1}(V) \subset bCl(f^{-1}(V))$ and $x \notin f^{-1}(Cl(V))$. Since f is subweakly b-continuous, $x \notin bCl(f^{-1}(V))$. Then, using Lemma 2.1(iii), we have $X \setminus bCl(f^{-1}(V))$ is a b-open set containing x but not y. By an argument similar to that of the above proof, it is shown that there exists a subset $W \in \mathcal{B}$ such that $X \setminus bCl(f^{-1}(W))$ is a b-open set containing y but not x. It follows that (X,τ) is b-T_1. **q.e.d**

Lemma 3.10 *Let (X,τ) be a topological space and A a subset of (X,τ). Then we have the following properties.*

(i) [14],[18, Proposition 3.9] (e.g. [1, Proof of Theorem 2.3(3)], [13, Lemma 2.2],[19, Lemma 3.2],[21, Lemma 5.2] , [25, Lemma 5.1]) *If $A \in \alpha O(X,\tau)$ and $U \in BO(X,\tau)$, then $U \cap A \in BO(A, \tau|A)$.*

(ii) [14] (e.g., [25, Lemma 5.2 (1)]) *If $A \in \alpha O(X,\tau)$ and $V \in BO(A,\tau|A)$ and then $V \in BO(X,\tau)$.*

(iii) [2, Proposition 2.4, Proposition 2.3(b)] *If $A \in \alpha O(X,\tau)$ and $U \in BO(X,\tau)$, then $U \cap A \in BO(X,\tau)$.*

(iv) [21, Lemma 5.3] *If* $B \subset A \subset X$ *and* $A \in \alpha O(X, \tau)$, *then* $(bCl(B)) \cap A = bCl_A(B)$, *where* $bCl_A(B) := \bigcap\{F \mid F$ *is* b*-closed in* $(A, \tau|A)$ *with* $B \subset F \subset A\}$.

Theorem 3.11 *If* $f : (X, \tau) \to (Y, \sigma)$ *is subweakly* b*-continuous and* $A \in \alpha O(X, \tau)$, *then* $f|A : (A, \tau|A) \to (Y, \sigma)$ *is subweakly* b*-continuous.*

Proof. Let $\mathcal{B}$ be an open base for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Then using Lemma 3.10 (iv) we have for $V \in \mathcal{B}$, $bCl_A((f|A)^{-1}\text{-}(Cl(V)) \subset A \cap bCl((f|A)^{-1}(V))$=$A \cap bCl(A \cap f^{-1}(V)) \subset A \cap bCl(f^{-1}(V)) \subset A \cap f^{-1}(Cl(V))$=$(f|A)^{-1}(Cl(V))$. Therefore, $f|A$ is subweakly b-continuous. **q.e.d.**

Theorem 3.12 *If* $f : (X, \tau) \to (Y, \sigma)$ *is subweakly* b*-continuous and* E *is an open subset of* (Y, σ) *with* $f(X) \subset E$, *then* $f : (X, \tau) \to (E, \sigma|E)$ *is subweakly* b*-continuous.*

Proof. Let $\mathcal{B}$ be an open base for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Then, the collection $\mathcal{C} := \{V \cap E \mid V \in \mathcal{B}\}$ is an open base for the relative topology $\sigma|E$ on E. Since E is open in (Y, σ), it is well known that $Cl(V) \cap E \subset Cl_E(V \cap E)$. Then, $bCl(f^{-1}(V \cap E)) \subset f^{-1}(Cl(V) \cap E) \subset f^{-1}(Cl_E(V \cap E))$; hence $f : (X, \tau) \to (E, \sigma|E)$ is subweakly b-continuous. **q.e.d.**

Theorem 3.13 *Let* $f : (X, \tau) \to (X, \tau)$ *be subweakly* b*-continuous and let* $A \subset X$ *such that* $f(X) \subset A$ *and* $f|A$ *is the identity function on* A. *Then, if* (X, τ) *is Hausdorff, then* A *is* b*-closed.*

Proof. Assume A is not b-closed. Let $x \in bCl(A) \setminus A$. Let $\mathcal{B}$ be an open base for the topology τ on X for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Since $x \notin A$, $f(x) \neq x$. Since (X, τ) is Hausdorff, there exist disjoint open sets V and W with $x \in V$, $f(x) \in W$ and $V \in \mathcal{B}$. Let $U \in BO(X, \tau; x)$. Then $x \in U \cap V$ which is b-open in (X, τ) by Lemma 3.10(iii). Since $x \in bCl(A)$, $(U \cap V) \cap A \neq \emptyset$. Let $y \in (U \cap V) \cap A$. Since $y \in A$, $f(y) = y \in V$ and hence $y \in f^{-1}(V)$. Therefore, $y \in U \cap f^{-1}(V)$ and hence $U \cap f^{-1}(V) \neq \emptyset$ and, using Lemma 2.1(i), we have $x \in bCl(f^{-1}(V))$. However, $f(x) \in W$ which is open and disjoint from V. So $f(x) \notin Cl(V)$ or, that is, $x \notin f^{-1}(Cl(V))$, which contradicts the assumption that f is also subweakly b-continuous. Therefore, A is b-closed. **q.e.d.**

Theorem 3.14 *If* $f_1 : (X, \tau) \to (Y, \sigma)$ *is* α*-continuous,* $f_2 : (X, \tau) \to (Y, \sigma)$ *is subweakly* b*-continuous, and* (Y, σ) *is Hausdorff, then the set* $A := \{x \in X \mid f_1(x) = f_2(x)\}$ *is* b*-closed in* (X, τ).

Proof. Let $x \in X \setminus A$. Then $f_1(x) \neq f_2(x)$. Let $\mathcal{B}$ be an open base for the topology σ on Y for which $bCl(f^{-1}(V)) \subset f^{-1}(Cl(V))$ for every $V \in \mathcal{B}$. Since (Y, σ) is Hausdorff, there exist disjoint open sets V and W with $f_1(x) \in V, f_2(x) \in W$ and $V \in \mathcal{B}$. Then $f_2(x) \notin Cl(V)$ and hence $x \notin f_2^{-1}(Cl(V))$. Then, since f_2 is subweakly b-continuous, $x \in X \setminus bCl(f_2^{-1}(V))$. Thus, $x \in f_1^{-1}(V) \cap (X \setminus bCl(f_2^{-1}(V))) \subset X \setminus A$. By Lemma 2.1(iii), $X \setminus bcl(f_2^{-1}(V))$ is b-open in (X, τ). Since $f_1^{-1}(V)$ is α-open in (X, τ), it follows from Lemma 3.10(iii) that the intersection of these sets is b-open in (X, τ). It follows from Lemma 2.1(ii) that $X \setminus A$ is b-open; and hence A is b-closed in (X, τ). **q.e.d.**

Recall that a subset A of a topological space (X, τ) is said to be b*-dense* [18] if $bcl(A) = X$.

Corollary 3.15 *Assume that* $f_1 : (X, \tau) \to (Y, \sigma)$ *is* α*-continuous,* $f_2 : (X, \tau) \to (Y, \sigma)$ *is subweakly* b*-continuous, and* (Y, σ) *is Hausdorff. If* f_1 *and* f_2 *agree on a* b*-dense set, then* $f_1 = f_2$.

Proof. Let $A := \{x \in X|\ f_1(x) = f_2(x)\}$ and let U be a b-dense set in (X, τ) on which f_1 and f_2 agree. Then, since $U \subset A$, we have $X = bCl(U) \subset bCl(A) = A$ (cf. Theorem 3.14) and hence $f_1 = f_2$. **q.e.d.**

Theorem 3.16 *If $f_j : (X, \tau) \to (Y_j, \sigma_j)$ is subweakly b-continuous for each $j \in \Lambda_m$ where $\Lambda_m := \{1, 2, ..., m\}(m > 1)$, then $f : (X, \tau) \to (\prod_{j=1}^{m} Y_j, \prod_{j=1}^{m} \sigma_j)$ given by $f(x) := (f_1(x), f_2(x), ..., f_m(x))$ is subweakly b-continuous.*

Proof. For each $j \in \Lambda$, let $\mathcal{B}_j$ be an open base for the topology on Y_j for which $bCl(f_j^{-1}(V_j)) \subset f_j^{-1}(Cl(V_j))$ for every $V_j \in \mathcal{B}_j$. Then, let $\mathcal{B} := \{\prod_{j=1}^{m} V_j | V_j \in \mathcal{B}_j (j \in \Lambda_m)\}$ be an open base for the topology $\prod_{j=1}^{m} \sigma_j$ on $\prod_{j=1}^{m} Y_j$. For every set $V := \prod_{j=1}^{m} V_j \in \mathcal{B}$, $bCl(f^{-1}(V))$=$bCl(\bigcap\{f_j^{-1}(V_j)|j \in \Lambda\}) \subset \bigcap\{bCl(f_j^{-1}(V_j))|\ j \in \Lambda_m\} \subset \bigcap\{f_j^{-1}(Cl(V_j))|j \in \Lambda_m\}$=$f^{-1}(\prod_{j=1}^{m}(Cl(V_j)))$=$f^{-1}(Cl(V))$. Thus, f is subweakly b-continuous. **q.e.d.**

Theorem 3.17 *If $f_j : (X_j, \tau_j) \to (Y_j, \sigma_j)$ is subweakly b-continuous for each $j \in \Lambda_m$ where $\Lambda_m := \{1, 2, ..., m\}(m > 1)$, then a function $\prod_{j=1}^{m} f_j : (\prod_{j=1}^{m} X_j, \prod_{j=1}^{m} \tau_j) \to (\prod_{j=1}^{m} Y_j, \prod_{j=1}^{m} \sigma_j)$ defined by $(\prod_{j=1}^{m} f_j)(x) := (f_1(x), f_2(x), ..., f_m(x))$ is subweakly b-continuous.*

Proof. For each $j \in A$, let $\mathcal{B}_j$ be an open base for the topology on Y_j for which $bCl(f_j^{-1}(V_j)) \subset f_j^{-1}(Cl(V_j))$ for every $V_j \in \mathcal{B}_j$. Let $\mathcal{B} := \{\prod_{j=1}^{m} V_j | V_j \in \mathcal{B}_j (j \in \Lambda_m)\}$ be an open base for the topology $\prod_{j=1}^{m} \sigma_j$ on $\prod_{j=1}^{m} Y_j$. For every set $V := \prod_{j=1}^{m} V_j \in \mathcal{B}$, we have that: $bCl((\prod_{j=1}^{m} f_j)^{-1}(V)) \subset \prod_{j=1}^{m} bCl((f_j)^{-1}(V_j)) \subset \prod_{j=1}^{m}(f_j)^{-1}(Cl(V_j))$ $=(\prod_{j=1}^{m} f_j)^{-1}(\prod_{j=1}^{m} Cl(V_j))$ $=(\prod_{j=1}^{m} f_j)^{-1}(Cl(\prod_{j=1}^{m} V_j))$ $=(\prod_{j=1}^{m} f_j)^{-1}(Cl(V))$. Thus, $\prod_{j=1}^{m} f_j$ is subweakly b-continuous. **q.e.d.**

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