FUZZY FILTERS OF A BCK-ALGEBRA

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Abstract. We study fuzzy filters of a BCK-algebra and characterize fuzzy s-prime filters and fuzzy maximal filters of a BCK-algebra.

1. Introduction

The fundamental concept of fuzzy sets was introduced by L. A. Zadeh [8] in 1965. It has been applied by many authors to study the fuzzification of some basic notions of a BCK-algebra (see for instance, [2]–[4] and [6]–[7]).

In this work, we introduce the notion of a fuzzy BCK-filter with values in a distributive lattice L. It generalizes the basic notion of a fuzzy BCK-filter with values in the unit interval $[0, 1]$ of real numbers, introduced and studied by Y. B. Jun et al. in [3]–[4]. In Section 2, we review a few basic definitions from the theory of BCK-algebras and fuzzy set logic and set up our notation for the development of this article. In Section 3, we define fuzzy s-prime filters and fuzzy maximal filters of a BCK-algebra and give their characterizations. We show that a fuzzy s-prime filter of a bounded commutative BCK-algebra X is determined by a prime filter of X and a prime element of L, and vice-versa. In particular, for any filter F of X, the characteristic function χ_F of F is a fuzzy s-prime filter of X if and only if F is a prime filter of X and 0 is a prime element of L.

2. Preliminaries

A *BCK-algebra* is a system $(X, \leq, 0)$ together with a binary operation denoted by juxtaposition such that the following axioms are satisfied for all x, y, and z in X:

(1) $(xy)(xz) \leq zy$,
(2) $x(xy) \leq y$,
(3) $x \leq x$,
(4) $0 \leq x$,
(5) $x \leq y$ and $y \leq x$ imply $x = y$,
(6) $x \leq y$ if and only if $xy = 0$.

The following assertions are true for any x, y, and z in a BCK-algebra X:

(I) $x0 = x$,
(II) $(xy)z = (xz)y$,
(III) $xy \leq x$,
(IV) $(xz)(yz) \leq xy$,
(V) $x \leq y$ imply $xz \leq yz$ and $zy \leq zx$.

A BCK-algebra X is said to be *commutative* if it satisfies the identity $x \wedge y = y \wedge x$ for all x and y in X, where $x \wedge y = y(yx)$. If a BCK-algebra X has a special element 1 such that $x \leq 1$ for all x in X, then 1 is called *unit* of X, and a BCK-algebra with unit is said to be *bounded.* We denote $1x$ by x^* and $(x^* \wedge y^*)^*$ by $x \vee y$ for any x and y in a

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bounded BCK-algebra X. The following assertions are true for any x and y in a bounded BCK-algebra X:

(1) $0^* = 1$ and $1^* = 0$,
(2) $x \leq y$ implies $y^* \leq x^*$,
(3) $x^*y^* \leq yx$,
(4) $(x^*)^* = x$.

A nonempty subset F of a BCK-algebra is said to be a *filter* if (1) $1 \in F$ and (2) $(x^*y^*)^* \in F$ and $y \in F$ imply $x \in F$ for all x and y in X. It is well-known that the identity $x^*y^* = yx$ holds for all x and y in a bounded commutative BCK-algebra X. Hence in a bounded commutative BCK-algebra X the condition (2) of the definition of a filter coincides with the condition $(2)'$: $(yx)^* \in F$ and $y \in F$ imply $x \in F$ for all x and y in X. A proper filter F of a bounded commutative BCK-algebra X is said to be a *prime filter* if $x \vee y \in F$ implies either $x \in F$ or $y \in F$, for all x and y in X. A proper filter F of X is said to be a *maximal filter* if $F \subseteq A \subseteq X$ implies either $F = A$ or $A = X$, for any filter A of X. If A is a nonempty subset of a bounded BCK-algebra X, then the set of all $x \in X$ satisfying $(\cdots((x^*a_1^*)a_2^*)\cdots a_{n-1}^*)a_n^* = 0$ for some $a_1, a_2, \ldots, a_n \in A$ is the minimal filter containing A [5]. It is called the filter generated by A and is denoted by $\langle A \rangle$. In particular, if $A = \{a\}$, then we will denote $\langle \{a\} \rangle$ simply by $\langle a \rangle$. See [5] for more details on filter theory of BCK-algebras.

In what follows, L will always denote, unless mentioned otherwise, a distributive lattice with a least element 0 and a greatest element 1. If X is the universe of discourse, all fuzzy subsets of X throughout this paper will be L-fuzzy subsets in the sense of Goguen [1], that is, maps from X to L. If L, in particular, is the unit interval of real numbers $[0, 1]$, then L-fuzzy subsets are fuzzy subsets in the usual sense [8]. However, for the sake of simplicity, we will write fuzzy subsets instead of L-fuzzy subsets. A *nonempty fuzzy subset* of X is a fuzzy subset of X which is not a constant map which assumes the value 0 of L. For any two fuzzy subsets λ and μ of X, the inequality $\lambda \leq \mu$ means that $\lambda(x) \leq \mu(x)$ for all $x \in X$. The symbols $\lambda \vee \mu$ and $\lambda \wedge \mu$ will mean the fuzzy subsets of X defined by $(\lambda \vee \mu)(x) = \lambda(x) \vee \mu(x)$ and $(\lambda \wedge \mu)(x) = \lambda(x) \wedge \mu(x)$ for all $x \in X$.

3. Fuzzy BCK-filters

In the sequel, X will denote a bounded commutative BCK-algebra with unit 1, unless mentioned otherwise.

Definition 1. *A nonempty fuzzy subset μ of X is said to be a fuzzy filter if* (a) $\mu(1) \geq \mu(x)$ *and* (b) $\mu(x) \geq \mu(yx)^* \wedge \mu(y)$ *for all x and y in X.*

Remark 3.1. *(1) If μ is a fuzzy filter of X and $x \leq y$, then $\mu(x) \leq \mu(y)$. In fact, $x \leq y$ implies $xy = 0$ which implies $(xy)^* = 0^* = 1$ and hence by part (a) of Definition 1 we get $\mu(xy)^* \geq \mu(x)$ which implies that $\mu(xy)^* \wedge \mu(x) = \mu(x)$. Thus it follows from part (b) of Definition 1 that $\mu(y) \geq \mu(xy)^* \wedge \mu(x) = \mu(x)$.*

(2) If F is a filter of X and $\alpha \leq \beta$ are two elements of L, then the fuzzy subset μ of X defined by

$$\mu(x) = \begin{cases} \beta & \text{if } x \in F \\ \alpha & \text{otherwise} \end{cases}$$

is a fuzzy filter of X. In fact, $1 \in F$ implies $\mu(1) = \beta$ and so the inequality $\mu(1) \geq \mu(x)$ holds for all x in X. Now, if $x \in F$, then the inequality $\mu(x) \geq \mu(yx)^* \wedge \mu(y)$ is obvious. If $x \notin F$, then either $(yx)^* \notin F$ or $y \notin F$, hence either $\mu(yx)^* = \alpha$ or $\mu(y) = \alpha$, and so $\mu(yx)^* \wedge \mu(y) = \alpha$. Thus it follows that the inequality $\mu(x) \geq \mu(yx)^* \wedge \mu(y)$ holds for all x and y in X.

(3) If F is a nonempty subset of X, then F is a filter of X if and only if the characteristic function χ_F of F is a fuzzy filter of X. In fact, it follows from part (2) above that χ_F is a fuzzy filter of X whenever F is a filter of X. Conversely, assume that χ_F is a fuzzy filter of X. Since F is nonempty, there exists an element x in F, and therefore $\chi_F(x) = 1$. It follows from part (1) above that $\chi_F(1) = 1$, and so $1 \in F$. Now, if $(yx)^*$ and y are in F, then the inequality $\chi_F(x) \geq \chi_F(yx)^* \wedge \chi_F(y) = 1$ implies that x is in F. Thus we have that F is a filter of X.

(4) If μ is a fuzzy filter of X, then $F = \{x \in X : \mu(x) = \mu(1)\}$ is a filter of X. In fact, it is clear that $1 \in F$. If $(yx)^*$ and y are in F, then $\mu(1) \geq \mu(x) \geq \mu(yx)^* \wedge \mu(y) = \mu(1)$, and so $x \in F$. Thus it follows that F is a filter of X.

Definition 2. *A nonconstant fuzzy filter π of X is said to be a fuzzy prime filter of X if $\pi(x \vee y) \leq \pi(x) \vee \pi(y)$ for all x and y in X.*

Definition 3. *A nonconstant fuzzy filter π of X is said to be a fuzzy s-prime filter of X if $\mu_1 \wedge \mu_2 \leq \pi$ implies either $\mu_1 \leq \pi$ or $\mu_2 \leq \pi$ for any two fuzzy filters μ_1 and μ_2 of X.*

We will show that every fuzzy s-prime filter is a prime filter (see Corollary 3.6 on page (4)), however, the converse may not be true even in the particular case when $L = [0, 1]$ (see, for instance, [7]).

Definition 4. *An element $\alpha \neq 1$ of L is said to be a prime element of L if $\alpha_1 \wedge \alpha_2 \leq \alpha$ implies either $\alpha_1 \leq \alpha$ or $\alpha_2 \leq \alpha$ for any two elements α_1 and α_2 of L.*

Lemma 3.2. [5] *A proper filter F of a bounded commutative BCK-algebra X is prime if and only if $A \cap B = F$ implies either $A = F$ or $B = F$ for any two filters A and B of X.*

Theorem 3.3. (a) *Let F be a prime filter of a bounded commutative BCK algebra X and α a prime element of L. Then the fuzzy subset of X defined by*

$$\pi(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{otherwise} \end{cases}$$

is a fuzzy s-prime filter of X.
(b) *Conversely any fuzzy s-prime filter of X can be obtained as above.*

Proof. (a) It is clear from part (2) of Remark 3.1 that π is a nonconstant fuzzy filter of X. If there exist two fuzzy filters μ and ν of X such that $\mu \not\leq \pi$ and $\nu \not\leq \pi$, then $\mu(x) \not\leq \pi(x)$ and $\nu(y) \not\leq \pi(y)$ for some x and y in X. It follows that $\pi(x) = \alpha$ and $\pi(y) = \alpha$, hence $x \notin F$ and $y \notin F$, and since F is a prime filter of X, we have that $x \vee y \notin F$, and so $\pi(x \vee y) = \alpha$. Because $x \leq x \vee y$ and $y \leq x \vee y$, it follows from part (1) of Remark 3.1 that $\mu(x) \leq \mu(x \vee y)$ and $\nu(y) \leq \nu(x \vee y)$, and hence $\mu(x) \wedge \nu(y) \leq \mu(x \vee y) \wedge \nu(x \vee y)$. Since α is a prime element of L, we have that $\mu(x) \wedge \nu(y) \not\leq \alpha$, and so it follows that $\mu(x \vee y) \wedge \nu(x \vee y) \not\leq \alpha = \pi(x \vee y)$ which implies that $(\mu \wedge \nu)(x \vee y) \not\leq \pi(x \vee y)$, and hence $\mu \wedge \nu \not\leq \pi$. Thus π is a fuzzy s-prime filter of X.

(b) First, we show that $\pi(1) = 1$. For if, $\pi(1) < 1$, then as π is nonconstant, we have that $\pi(a) < \pi(1)$ for some a in X. It follows from parts (3) and (4) of Remark 3.1 that the fuzzy subset μ of X defined by

$$\mu(x) = \begin{cases} 1 & \text{if } \pi(x) = \pi(1) \\ 0 & \text{otherwise} \end{cases}$$

is a fuzzy filter of X. Define the fuzzy filter ν of X by $\nu(x) = \pi(1)$ for all x in X. Clearly, $\mu \wedge \nu \leq \pi$, but $\mu(1) = 1 > \pi(1)$ and $\nu(a) = \pi(1) > \pi(a)$ imply that $\mu \not\leq \pi$ and $\nu \not\leq \pi$, a contradiction. Hence $\pi(1) = 1$.

Next, we show that the nonconstant fuzzy filter π assumes exactly two values by establishing that if a and b are two elements of X such that $\pi(a) < 1$ and $\pi(b) < 1$, then $\pi(a) = \pi(b)$. Indeed, define the fuzzy ideals μ and ν of X by

$$\mu(x) = \begin{cases} 1 & \text{if } x \in \langle a \rangle \\ 0 & \text{otherwise} \end{cases}$$

and $\nu(x) = \pi(a)$ for all x in X. If $x \notin \langle a \rangle$, then $(\mu \wedge \nu)(x) = 0 \le \pi(x)$. If $x \in \langle a \rangle$, then there exists a positive integer n such $(\cdots((x^* \underbrace{a^*)a^*)\cdots)a^*}_{n} = 0$. Hence $(\cdots((x^* \underbrace{a^*)a^*)\cdots)a^*}_{n-1} \le a^*$ which implies $a \le ((\cdots((x^* \underbrace{a^*)a^*)\cdots)a^*}_{n-1})^*$ and so by part (1) of Remark 3.1 we have $\pi(a) \le \pi(((\cdots((x^* \underbrace{a^*)a^*)\cdots)a^*}_{n-1})^*)$. So $\pi(a) = \pi(a) \wedge \pi(((\cdots((x^* \underbrace{a^*)a^*)\cdots)a^*}_{n-1})^*) = \pi(a) \wedge \pi((a((\cdots((x^* \underbrace{a^*)a^*)\cdots)a^*}_{n-2})^*)^*)$, and since π is a fuzzy filter, therefore, it follows that $\pi(a) \le \pi(((\cdots((x^* \underbrace{a^*)a^*)\cdots)a^*}_{n-2})^*)$. Continuing in this way we get $\pi(a) \le \pi((x^*)^*) = \pi(x)$, and so $(\mu \wedge \nu)(x) = \nu(x) = \pi(a) \le \pi(x)$. It follows that $\mu \wedge \nu \le \pi$. But $\mu(a) = 1 > \pi(a)$ implies that $\mu \not\le \pi$, and so $\nu \le \pi$ since π is a fuzzy s-prime filter of X. Hence $\nu(b) \le \pi(b)$ which implies that $\pi(a) \le \pi(b)$. Similarly, one can show that $\pi(b) \le \pi(a)$, and hence $\pi(a) = \pi(b)$. Thus it follows that the fuzzy s-prime filter π of X assumes exactly two values.

Let $\alpha \in L$ denote the value of π other than 1, and let $F = \{x \in X : \pi(x) = 1\}$. Since π is a nonconstant fuzzy filter of X and $\pi(1) = 1$, it follows from part (4) of Remark 3.1 that F is a proper filter of X. Observe that for any x in X we have

$$\pi(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{otherwise} \end{cases}$$

and so it is sufficient to show that F is a prime filter of X and α is a prime element of L. Since π is a fuzzy s-prime filter of X, and $a \le \alpha$ if and only if $\lambda_a \le \pi$ for any element a of L and the constant map λ_a with value a, it follows that α is a prime element of L. If A and B are two filters of F such that $A \cap B = F$, then $\chi_A \wedge \chi_B = \chi_{A \cap B} = \chi_F \le \pi$. Since π is a fuzzy s-prime filter of X, therefore, either $\chi_A \le \pi$ or $\chi_B \le \pi$, and so $A \subseteq F$ or $B \subseteq F$, but since $F \subseteq A$ and $F \subseteq B$, it follows that either $A = F$ or $B = F$. This completes the proof. □

Corollary 3.4. *Let F be a subset of X. Then the characteristic function χ_F of F is fuzzy s-prime filter of X if and only if F is a prime filter of X and 0 is a prime element of L.*

Corollary 3.5. *Let π be a fuzzy subset of X, and let L be a chain. Then π is a fuzzy s-prime filter of X if and only if there exist a prime filter F of X and an element $\alpha < 1$ such that*

$$\pi(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{otherwise.} \end{cases}$$

In particular, if $L = [0, 1]$. Then a fuzzy subset π of X is its fuzzy s-prime filter if and only if there exist a prime filter F of X and an element $\alpha \in [0, 1)$ such that

$$\pi(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{otherwise.} \end{cases}$$

Corollary 3.6. *Every fuzzy s-prime filter of X is a prime filter of X.*

Proof. Let π be a fuzzy s-prime filter of X. By Theorem 3.3, there exists a prime filter F of X and a prime element α of L such that

$$\pi(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{otherwise.} \end{cases}$$

If $x \vee y \notin F$, then clearly $\pi(x \vee y) \leq \pi(x) \vee \pi(y)$. If $x \vee y \in F$, then, since F is a prime filter of X, we have that either $x \in F$ or $y \in F$. Hence either $\pi(x) = 1$ or $\pi(y) = 1$ which implies $\pi(x) \vee \pi(y) = 1$, and so again $\pi(x \vee y) \leq \pi(x) \vee \pi(y)$. Thus π is a prime filter of X. □

Definition 5. *A nonconstant fuzzy filter μ of X is said to be its fuzzy maximal filter if $\mu \leq \nu$ implies either $\mu = \nu$ or ν is a constant fuzzy filter of X, for any filter ν of X.*

Definition 6. *An element $\alpha \neq 1$ of L is said to be its dual atom if $\alpha \leq \beta$ implies either $\alpha = \beta$ or $\beta = 1$, for any element β of L.*

Theorem 3.7. (a) *Let F be a maximal filter of a bounded commutative BCK-algebra X and α a dual atom of L. Then the fuzzy subset of X defined by*

$$\mu(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{otherwise} \end{cases}$$

is a fuzzy maximal filter of X.
(b) *Conversely any fuzzy maximal filter of X can be obtained as above.*

Proof. (a) It is clear from part (2) of Remark 3.1 that μ is a nonconstant fuzzy filter of X. If ν is a nonconstant fuzzy filter of X such that $\mu \leq \nu$, then $F \subseteq \{x \in X : \nu(x) = 1\} \neq X$, and hence $F = \{x \in X : \nu(x) = 1\}$ since F is a maximal filter of X. Clearly values of μ and ν agree on F, and if $x \notin F$, then $\alpha = \mu(x) \leq \nu(x) < 1$, and so $\mu(x) = \nu(x)$ since α if a dual atom of L. Hence $\mu = \nu$. Thus it follows that μ is a fuzzy maximal filter of X.

(b) First we show that $\mu(1) = 1$. For if, $\mu(1) < 1$, then it follows from parts (2) and (4) of Remark 3.1 that the fuzzy subset ν of X defined by

$$\nu(x) = \begin{cases} 1 & \text{if } \mu(x) = \mu(1) \\ \mu(1) & \text{otherwise} \end{cases}$$

is its fuzzy filter. Clearly, $\mu \leq \nu$ and ν is nonconstant, so the maximality of μ implies that $\mu = \nu$, and so $\mu(1) = 1$, a contradiction. Hence $\mu(1) = 1$.

Next, we show that the nonconstant fuzzy filter μ assumes exactly two values by establishing that if a and b are two elements of X such that $\mu(a) < 1$ and $\mu(b) < 1$, then $\mu(a) = \mu(b)$. Indeed, first observe that since L is a distributive lattice, therefore, the fuzzy subset $\mu \vee \beta$ of X defined by $(\mu \vee \beta)(x) = \mu(x) \vee \beta$ for all x in X is its fuzzy filter. Now $(\mu \vee \mu(a))(1) = \mu(1) \vee \mu(a) = 1$ and $(\mu \vee \mu(a))(a) = \mu(a)$ imply that $\mu \vee \mu(a)$ is a nonconstant fuzzy filter of X. Hence the inequality $\mu \leq \mu \vee \mu(a)$ and the maximality of μ imply that $\mu = \mu \vee \mu(a)$. It follows that $\mu(b) = (\mu \vee \mu(a))(b) = \mu(b) \vee \mu(a)$ and hence $\mu(a) \leq \mu(b)$. Similarly, one can show that $\mu(b) \leq \mu(a)$, and hence $\mu(a) = \mu(b)$. Thus it follows that the fuzzy maximal filter μ assumes exactly two values.

Let α denote the value of μ other than 1, and let $F = \{x \in X : \mu(x) = 1\}$. Since μ is a nonconstant fuzzy filter of X and $\mu(1) = 1$, it follows from part (4) of Remark 3.1 that F is a proper filter of X. Observe that for any x in X we have

$$\mu(x) = \begin{cases} 1 & \text{if } x \in F \\ \alpha & \text{otherwise} \end{cases}$$

and so it is sufficient to show that F is a maximal filter of X and α is a dual atom of L. If β is an element of L such that $\alpha \leq \beta < 1$, then

$$\nu(x) = \begin{cases} 1 & \text{if } x \in F \\ \beta & \text{otherwise} \end{cases}$$

is a nonconstant fuzzy filter of X and $\mu \leq \nu$. Hence the maximality of μ implies that $\mu = \nu$, therefore, $\alpha = \beta$, and so it follows that α is a dual atom of L. If A is a filter of X such that $F \subset A \subseteq X$, then

$$\omega(x) = \begin{cases} 1 & \text{if } x \in A \\ \alpha & \text{otherwise} \end{cases}$$

is a fuzzy filter of X and $\mu < \omega$. Since μ is a fuzzy maximal filter of X, therefore, ω is constant, and so $A = X$. This completes the proof. □

Corollary 3.8. *If $L = [0, 1]$, then X has no fuzzy maximal filters.*

In Example 3.12 on page (6) a fuzzy s-prime filter of a bounded commutative BCK-algebra is constructed with values in $L = [0, 1]$. It follows that a fuzzy s-prime filter may not be fuzzy maximal filter even in the particular case when $L = [0, 1]$. However, we will show that every fuzzy maximal filter of a bounded commutative BCK-algebra is also its fuzzy s-prime filter (see Corollary 3.11 further below).

Corollary 3.9. *Let a and b be two real numbers such that $a < b$, and let S be a subset of $(-\infty, a)$ with a least element. If $L = S \cup \{a, b\}$, then a fuzzy subset μ of X is its fuzzy maximal filter if and only if there exists a maximal filter F of X such that*

$$\mu(x) = \begin{cases} 1 & \text{if } x \in F \\ a & \text{otherwise.} \end{cases}$$

It is shown in [5] that for any filter F of a bounded commutative BCK-algebra X and $x \in X \setminus F$, there exists a prime filter A of X such that $F \subseteq A$ and $x \notin A$. Thus the following result follows immediately.

Proposition 3.10. *Every maximal filter of a bounded commutative BCK-algebra is also its prime filter.*

Since every dual atom of a distributive lattice is also its prime element, therefore, we have the following result.

Corollary 3.11. *Every fuzzy maximal filter of a bounded commutative BCK-algebra is also its fuzzy s-prime filter.*

Let us consider the following example to demonstrate a few immediate applications of our results.

Example 3.12. *Let X be the set $\{0, 1, 2, 3\}$ with binary operation defined by the following table.*

	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

Then X is a bounded commutative BCK-algebra with unit 3 (cf. [7]). It is a routine matter to verify that $F = \{1, 2, 3\}$ is a filter of X. Clearly, the filter F is maximal, as well. Hence, by Proposition 3.10, F is a prime filter of X.

If $L = [0, 1]$, it follows from Corollary 3.5 that the characteristic function χ_F of F is a fuzzy s-prime filter of X.

If $L = \{0, 0.5, 1\}$, it follows from Corollary 3.9 that the fuzzy subset μ of X, defined by $\mu(0) = 0.5$ and $\mu(x) = 1$ otherwise, is a fuzzy maximal filter of X.

Now we remark that Corollary 3.8 suggests in order to initiate a study of fuzzy maximal filters of a BCK-algebra it is important to generalize the traditional notion of a fuzzy filter to one with its truth values in a distributive lattice.

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