OPTIMAL FACILITY LOCATION PROBLEM UNDER POSSIBILITY CHANCE CONSTRAINT CONDITIONS AND BARRIERS

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Abstract

We consider the following problem: 1) There are demand points and possible construction sites in an urban area with some barriers. We adopt rectilinear distance. 2) We construct two facilities, one is welcome facility and the other obnoxious facility We call welcome facility as A and obnoxious facility as B. Two facilities A and B can be constructed at the same site or constructed separately, that is, at two different sites. We assume that each construction cost of A and B is a random variable with fuzzy mean respectively and construction cost of both facilities simultaneously as a same site is also random variable with fuzzy mean. These are distributed according to normal distributions with fuzzy means. 3) The probability that total construction cost becomes below budget f should not be less than the fixed probability level. α and further the possibility that this chance constraint holds should be not less than the fixed level β. Under this possibility chance constraint f should be minimized. 4) We consider three criteria, (a) maximum distance from the construction site of A to all demand points to be minimized, (b) minimum distance from the construction site of B to all demand points to be maximized, (c) budget to be minimized. Since usually there exists no site optimizing three criteria at a time, we seek non-dominated solution after definition of non-domination. Finally, we conclude results and discuss further research problems.

1 Introduction There are huge amount of papers regarding facility location problem after Weber has published his paper [8] (so called Weber problem). Hamacher et al. ([2]) tried to classify these papers by introducing similar codes to classify queueing and scheduling models . For rectilinear distance, we should refer to [1] as a classic but successful model and an efficient algorithm due to geometrical approach. Further for a discrete location problem, refer to review paper [7]. In this paper we consider multi-facility case as one possibility based on rectilinear distance. That is, two types of facilities, welcome facility , the other obnoxious one are constructed. We call welcome facility as A and obnoxious facility as B. Two facilities A and B can be constructed at the same site or constructed separately at two different sites. Construction costs of A and B are random variables with fuzzy means. Section 2 formulates the facility location problem with the tri-criteria under above prominent features. Section 3 proposes a solution procedure to seek non-dominated solutions after the definition of non-domination. Finally, section 4 summarizes the results and discusses further research problem.

2 Problem formulation We consider the following problem:
(1) There are m demand points : $D_i = (a_i, b_i)$ for $i = 1, 2, ..., m$ and r possible construction sites, FP_j for $j = 1, 2, ..., n$ in an urban area $X = \{(x, y)|0 \leq x \leq p_0.0 \leq y \leq q_0\}$ with some rectangular barriers
$\mathbf{B_k} = \{(x, y)|B_k^1 < x < B_k^2, B_k^3 < y < B_k^4\}, k = 1, 2, ..., s$ Facilities A and B can be constructed in these blocks . Barrier means we cannot pass it inside and so in some case we must make a detour. We adopt rectilinear distance which is used often in an urban area. That is, rectilinear distance between points $P = (a, b)$ and $Q = (c, d)$ is $|a - c| + |b - d|$.
(2) We construct two facilities, one is welcome facility (that is, maximum distance to demand points should be minimized), the other is obnoxious one (that is, minimum distance to demand points should be maximized). We call welcome one as A and obnoxious one as B and two facilities A, B can be constructed at the same site or constructed separately, that is, at two different sites. For each possible construction site FP_j, we assume that each construction cost of A, B is a random variable CA_j, CB_j with fuzzy mean respectively and construction cost of both facilities simultaneously at a same site is also random variable CS_j with fuzzy mean . CA_j is distributed according to the normal distribution with fuzzy mean M_{1j} and variance σ_{1j}^2, CB_j according that with fuzzy mean M_{2j} and variance σ_{2j}^2, and CS_j according to fuzzy mean M_{3j} and variance σ_{3j}^2. We assume that they are independent each other. 縲 Note that if two facilities are constructed at different sites, the total construction cost is the sum of the construction cost of A and that of B. Each mean M_{uj} is a L fuzzy number with $L(\dfrac{t - m_{uj}}{\sigma_{uj}}), u = 1, 2, 3$.
(3) The probability that total construction cost becomes below budget f should be not less than the fixed probability level α and f should be minimized where we assume that $\alpha > \frac{1}{2}$. For A, B, separately constructed case at j, this probabilistic condition is

$$\Pr\{CA_j \leq f\} \geq \alpha \Leftrightarrow \Pr\{\frac{CA_j - m_{1j}}{\sigma_{1j}} \leq \frac{f - m_{1j}}{\sigma_{1j}}\} \geq \alpha \Leftrightarrow f \geq m_{1j} + K_\alpha \sigma_{1j}$$

where K_α is a α percentile points of the cumulative distribution function of the standard normal distribution since $\dfrac{CA_j - m_{1j}}{\sigma_{1j}}$ is a random variable according to the standard normal distribution. Similarly done, for the case of separate construction of B, we have the following deterministic equivalent condition as $f \geq m_{2j} + K_\alpha \sigma_{2j}$and for the case that both A and B are constructed at the same site , corresponding deterministic equivalent condition is

$f \geq m_{3j} + K_\alpha \sigma_{3j}$. Summarizing we have

$f \geq m_{1j}+K_\alpha\sigma_{1j}(\mathrm{A}: site\ \ j),\ \ f \geq m_{2j}+K_\alpha\sigma_{2j}(\mathrm{B}: site\ \ j),\ \ f \geq m_{3j}+K_\alpha\sigma_{3j}(\mathrm{both A, B}: site\ \ j)$

but if A, B are constructed at different possible sites FP_i, FP_j respectively, the budget constraint is

$$f \geq (m_{1i} + m_{2j}) + K_\alpha\sqrt{\sigma_{1i}^2 + \sigma_{2j}^2 + 2\sigma_{1i2j}}$$

where σ_{1i2j} is a covariance between CA_i and CB_j since $CA_i + CB_j$ is a random variable according to the normal distribution with mean $(m_{1j} + m_{2j})$ and variance $\sigma_{1i}^2 + \sigma_{2j}^2 + 2\sigma_{1i2j}$.
(4) We consider three criteria, that is, maximum distance from the construction site of A to all demand points to be minimized, minimum distance from the construction site of B to all demand points to be maximized and budget to be minimized. Let $d(i, j)$ be the distance between demand point $D_i, i = 1, 2, ..., m$ and possible construction site $FP_j, j = 1, 2, ..., n$. These are calculated using some algorithm (for example, matrix algorithm using path algebra) of the shortest path problem on the following networks $N(V, E)$ (refer to [4]):

$V = \{D_1, D_2., \cdots, D_m, (B_1^1, B_1^3), (B_1^1, B_1^4), (B_1^2, B_1^3), (B_1^2, B_1^4), \cdots (B_i^1, B_i^3), (B_i^1, B_i^4), (B_i^2, B_i^3), (B_i^2, B_i^4),$
$\cdots (B_s^1, B_s^3), (B_s^1, B_s^4), (B_s^2, B_s^3), (B_s^2, B_s^4), FP_1, FP_2, \cdots, FP_n\}(= \{v_1, v_2, \cdots, v_m, v_{m+1}, \cdots, v_{m+4s},$

$v_{m+4s+1}, \cdots, v_{m+4s+n}\})$

and E consists of edges corresponding to visible pairs between two vertices in V where length of each edge is a rectilinear distance between corresponding pair of verticies . Two points P^1, P^2 are called visible each other if there exists a route connecting two points using only horizontal line segment and vertical line segment not passing through some barriers without detours. .Otherwise we call $P^1 and P^2$ as invisible. In an invisible case we cannot connect two points by horizontal line segment and vertical line segment without detour like Figure 1.)

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Fig.1 An invisible point pair

For each edge, the rectilinear distance between corresponding vertices is attached as a length. Then the first criterion is $d_A(j) = \max\{d(i, j)|i = 1, 2, ..., m\}$ and $d_A(j)$ should be minimized about j=1,2,...,n. The second criterion is $d_B(j) = \min\{d(i, j)|i = 1, 2, ..., m\}$ and $d_B(j)$ should be maximized about j=1,2,...,n. The third criterion is minimum budget F under the above deterministic equivalent inequality, that is,

$$F = \min\{m_{1j_A} + m_{2j_B} + K_\alpha\sqrt{\sigma_{1j_A}^2 + \sigma_{2j_B}^2 + 2\sigma_{1j_A 2j_B}}, m_{3j_C} + K_\alpha\sigma_{3j_C}\}$$

where j_A: the site of facility A, j_B: the site of facility B if separately constructed and J_C is the site that both A, B are constructed at the same site j_C. However if we assume that

$$m_{1i} + m_{2j} + K_\alpha\sqrt{\sigma_{1i}^2 + \sigma_{2j}^2 + 2\sigma_{1i2j}} > \max\{m_{3i} + K_\alpha\sigma_{3i}, m_{3j} + K_\alpha\sigma_{3j}\}$$

for any pair of (i, j), $F = \min\{m_{3i} + K_\alpha\sigma | i = 1, 2, ..., n\}$. Since usually there exists no site optimizing tri-criteria at a time and so we seek some non-dominated solutions for the above model (1)-(4) after definition of non-domination in the next section.

3 Solution Procedure First we define a solution vector $V^X = (V_1^X, V_2^X, V_3^X)$ corresponding to a solution X where X is denoted as $X = (j_A^X, j_B^X)$ where J_A^X, j_B^X are construction sites of A and that of B respectively. Therefore

$$V_1^X = \max\{d(i, j_A^X) | i = 1, 2, ..., m\}, V_2^X = \min\{d(i, j_B^X) | i = 1, 2, ..., m\}$$

$$V_3^X = \begin{cases} m_{1j_A^X} + m_{2j_B^X} + \sqrt{\sigma_{1j_A^X}^2 + \sigma_{2j_B^X}^2 + \sigma_{12j_A^X j_B^X}^2} & (j_A^X \neq j_B^X) \\ m_{3j_A^X} + K_\alpha\sigma_{j_A^X} & (j_A^X = j_B^X) \end{cases}$$

Non-dominated Solution

For solutions X_1, X_2, if

$V_1^{X_1} \leq V_1^{X_2}$, $V_2^{X_1} \geq V_2^{X_2}$, $V_3^{X_1} \leq V_3^{X_2}$ and $V^{X_1} \neq V^{X_2}$, then we call X_1 dominates X_2. If there exists no solution dominating solution X, then X is called non-dominated solution. We seek some non-dominated solutions. Note that usually $\min\{d_A(j) | j = 1, 2, ..., n\} \leq \max\{d(i, j_C) | i = 1, 2, ..., m\}$ and $\max\{d_B(j) | j = 1, 2, ..., n\} \geq \min\{d(i, j_C) | i = 1, 2, ..., m\}$ hold where j_C is the minimizer of $\min\{M_{3j} + K_\alpha\sigma_{3j} | j = 1, 2, ..., n\}$.

Therefore first we check whether it holds that $\min\{d_A(j) | j = 1, 2, ..., n\} = \max\{d(i, j_C) | i = 1, 2, ..., m\}$. and $\max\{d_B(j) | j = 1, 2, ..., n\} = \min\{d(i, j_C) | i = 1, 2, ..., m\}$.. If so, the optimal solution is to construct both facilities A, B at the same possible site j_C as a multi-facility. Otherwise (usually this case holds), we seek some non-dominated solution as below (5)-(7).

(5) First of all, above solution constructing the multi-facility at possible site FP_{j_c} is a non-dominated solution (if minimizer $j_C$is not unique, we must check the non-domination and choose non-dominated one or ones.

(6) We find the minimizer j_A of $\min\{d_A(j) | j = 1, 2, ..., n\}$ and maximizer j_B of $\max\{d_B(j) | j = 1, 2, ..., n\}$.Then solution that facility A is constructed at j_A and B at j_B is a non-dominated solution. Of course, if j_A or j_B is not unique, we check these solutions about non-domination and choose non-dominated one or ones.

(7)We consider the weighted convex sum of $d_A(j)$ and $d_B(j)$, that is, $W(j) = w_1 d_A(j) + w_2 d_B(j), w_1, w_2 > 0, w_1 + w_2 = 1$ and find the minimizer j_W Then a solution that both A and B are constructed at the site j_W as a multi-facility is non-dominated one. Again if j_W is not unique, then check the non-domination and choose non-dominated one or ones.

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4 Conclusion This paper considered construction of two facilities simultaneously at different site or at a same site as a multi-facility under the stochastic construction costs. Here we considered a finite possible construction sites but following are left further research problems.

(8) As for more suitable criteria, we should consider environmental load, especially for obnoxious facility.

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